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# A Conjugate Gradient Based Algorithm to Minimize the Sidelobe Level of Planar Arrays with Element Failures

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Element failures increase the sidelobe power level of an array. Reconfiguring the amplitude and phase of the remaining elements can partially compensate for the failed elements and thus reduce the sidelobe level. The result of this investigation is an algorithm that yields the reconfigured distribution by minimizing the ratio of the average peak sidelobe power level to the power in the mainbeam, taking into account the defective elements. The minimization of this nonlinear function is carried out via a conjugate gradient method. The algorithm is applied to the synthesis of sum and difference patterns of planar arrays.


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# Preface

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# I. Introduction

An antenna array is composed of many elements whose excitation amplitude and phase can be individually adjusted to yield a desired pattern. If all array elements operate properly, then well-known analytic techniques can be used to find the optimum amplitude and phase of each element, to yield a given beamwidth and sidelobe level. However, if any elements fail, then no analytic means exists to find the aperture distribution that compensates for the degradation of the pattern. Element failures destroy symmetry and cause sharp variations in the field intensity across the array aperture, increasing the sidelobe level of the power pattern. The simplest solution to this problem is to increase the taper of the array distribution in order to lower the sidelobes back to the design level. Unfortunately, this solution has several disadvantages. First, as the design sidelobe level for an array decreases, the sensitivity of the sidelobe level to a perturbation in an excitation value increases. Second, changing the design taper may broaden the main beam to an unacceptable level. Finally, merely increasing the taper does not compensate for lack of symmetry or smooth out sharp field variations. Compensation for the defective elements can be achieved, by numerically finding the excitation of each nondefective element that optimizes some function. This function must take into account the location of the failed elements.

The approach used in this study is to reformulate the optimization problem to take into account the element failures. An algorithm is developed that synthesizes the amplitude and phase of each nondefective element in order to produce the lowest sidelobe level for a given beamwidth. This algorithm can also be used to find the element failure limits of any array. More specifically, the algorithm can find the maximum number of elements beyond which it is impossible to recover lost performance. Also, the loss in performance can be determined by the geometric arrangement of the failed elements. The numerical implementation of the synthesis algorithm involves the minimiza-

tion of a nonlinear function, which is the ratio of the sum of a set of peak sidelobe power points to the power in the main beam. This minimization will be carried out via a conjugate gradient method.

## II. Power Pattern With Element Failures

This analysis neglects the element factor and considers only the array factor. No mutual coupling effects are modeled. Failed elements are deleted from the array factor. The $e^{j\omega t}$ time convention is understood, where $\omega$ is the angular frequency of the excitation with a freespace wavelength of $\lambda_0$. Letting $N$ denote the number of nonfailed elements, the array factor (AF) is given by

$$\mathrm{AF} = \sum_{n=1}^{N} I_n e^{j(k_x x_n + k_y y_n)} \tag{1}$$

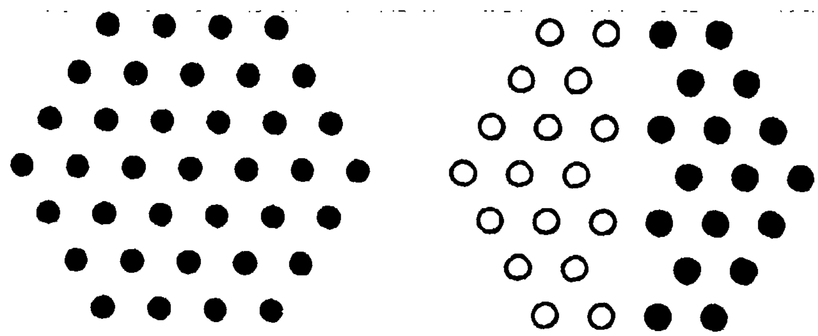
where

$$k_x = k_0 \sin(\theta)\cos(\phi) \tag{2}$$

$$k_y = k_0 \sin(\theta)\sin(\phi) \tag{3}$$

and $k_0 = 2\pi/\lambda_0$. The excitation of the $n$th element, denoted by $I_n$, can be expressed by the complex quantity $I_n = u_n + jv_n$, where $\sqrt{u_n^2 + v_n^2}$ yields the amplitude and $\tan^{-1}(v_n/u_n)$ yields the phase angle. The power pattern is proportional to the square of the magnitude of AF.

The array is assumed to operate in either sum or difference mode. The sum mode produces a peak in the pattern in the broadside direction and the difference mode produces a null in the broadside direction. A possible element configuration to generate both types of patterns is shown in Fig. 1. Sum mode operation allows all 37 elements to be independently controlled, as shown in Fig. 1(a). The same array operating in difference mode has only 17 independently controlled elements, as shown in Fig. 1(b). This is because half of the array elements are excited with their phase opposite from that of the corresponding elements symmetric with respect to the center of the array. If an element fails, then the symmetric counterpart is deactivated. This ensures a stationary beam null and a symmetric beam slope, independent of the synthesis technique used.



(a) (b)

Fig. 1. Independently Controlled Elements for (a) Sum Pattern and (b) Difference Pattern

The synthesis of the excitation values has three possible combinations: synthesizing with amplitude only, synthesizing with phase only, and synthesizing with both amplitude and phase. It is well known that synthesizing the lowest possible sidelobe level for a given beamwidth over a symmetric aperture yields a uniform phase and a nonuniform amplitude distribution. However, if the failed elements yield a nonsymmetric array distribution, then the optimal phase may not be constant. Phase synthesis with a constant amplitude was found by Deford and Gandhi (Ref. 1) to require an extremely large number of elements to yield low sidelobes. A similar analysis by Peters (Ref. 2) found that phase-only synthesis was not very effective in reducing sidelobes for fixed-amplitude arrays. Therefore, both amplitude and phase synthesis would seem to be required for the generation of a sum pattern. Since the difference pattern preserves the symmetry, amplitude-only synthesis is required. The sum power pattern is given by

$$P(\theta, \phi, u, v) = \left[\sum_{n=1}^{N} [u_n \cos(\psi_n) - v_n \sin(\psi_n)]\right]^2 + \left[\sum_{n=1}^{N} [u_n \sin(\psi_n) + v_n \cos(\psi_n)]\right]^2 \quad (4)$$

where $\psi_n = k_x x_n + k_y y_n$ and the difference power pattern may be written as

$$P(\theta, \phi, u) = \left[\sum_{n=1}^{N} u_n \sin(\psi_n)\right]^2 . \quad (5)$$

Note that the amplitude is allowed to be negative on the difference pattern, since that involves only a simple phase shift.

# III. Minimizing Nonlinear Functions

Equations (4) and (5) indicate that the power pattern of the array is a nonlinear function of the amplitude and phase of the excitation. Minimizing the peak sidelobe level will require a mathematical technique that can handle an nonlinear function. A multivariable quadratic function can be minimized by the conjugate gradient algorithm developed by Hestenes and Stiefel (Ref. 3). An arbitrary nonlinear function can also be minimized by the same algorithm if a suitable quadratic approximation can be found. Although several variations are given in the literature, all have the same two major steps discussed by Hestenes (Ref. 4). First, the function must be approximated by a quadratic truncation of a Taylor series expanded around an initial estimate of the solution. Second, this quadratic is minimized by the conjugate gradient algorithm to obtain a new estimate. This new estimate is in turn used as the expansion point for a new quadratic approximation. Since the quadratic approximation is valid only near the point of expansion, finding a minimum of this quadratic is the same as finding a minimum of the original function only if both have the same minimum. Therefore, the algorithm must be restarted so that the quadratic approximation can ultimately be expanded around a point that approaches the minimum of the original function. The one-dimensional case is illustrated in Fig. 2. Given the function $f(s)$ and the initial guess $s_0$, the quadratic $q_0(s)$ is formed and given by

$$q_0(s) = f(s_0) + f'(s_0)(s - s_0) + \frac{1}{2}f''(s_0)(s - s_0)^2. \tag{6}$$

The minimum of this quadratic is denoted by $s_1$ and is given by the Newton estimate

$$s_1 = s_0 - \frac{f'(s_0)}{f''(s_0)}. \tag{7}$$

The estimate $s_1$ is then used to compute $q_1(s)$. The iteration then continues until the minimum of the quadratic approximation coincides with the minimum of the original function.

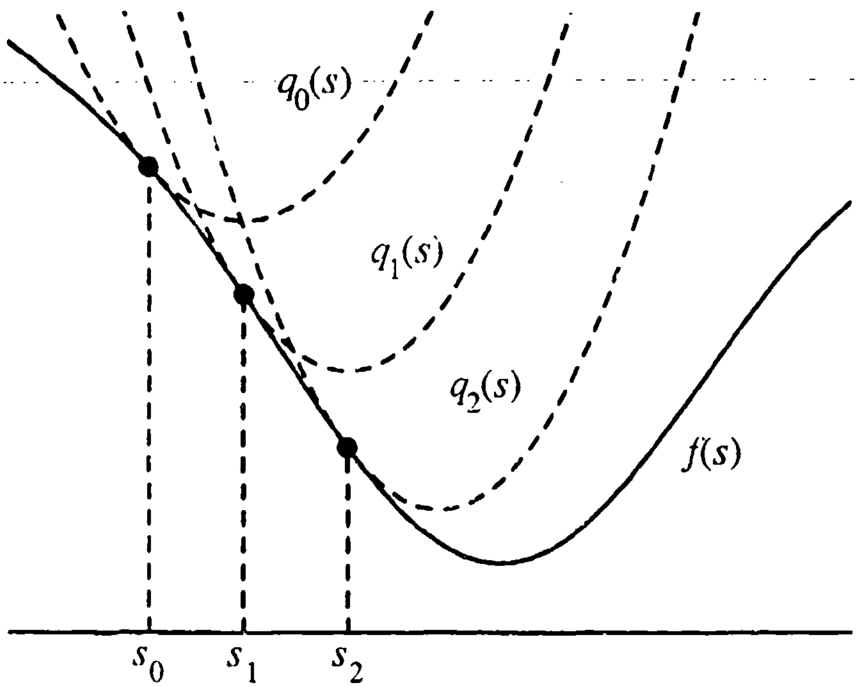



Fig. 2. One-Dimensional Minimization

The principles of the one-dimensional minimization apply also to multidimensional functions. For example, consider the function $f(u, v)$, where $u$ and $v$ are $N \times 1$ column vector variables. The quadratic approximation may be written in matrix form as

$$f(u,v) = f(u^1, v^1) + [\gamma - \gamma^1]^T G + \frac{1}{2}[\gamma - \gamma^1]^T H [\gamma - \gamma^1] \tag{8}$$

where $T$ denotes the transpose and the components are defined as follows. The vector $\gamma$ is a $2N \times 1$ column vector defined by $\gamma = [u \quad v]^T$. The gradient, denoted by $G$, is a $2N \times 1$ column vector evaluated at $(u^1, v^1)$ and is given by $G = [G_u \quad G_v]^T$, where each subvector has the form

$$G_u = [\frac{\partial f}{\partial u_1} \quad \frac{\partial f}{\partial u_2} \quad \cdots \quad \frac{\partial f}{\partial u_N}]. \tag{9}$$

The Hessian, denoted by $H$, is a $2N \times 2N$ matrix whose elements are evaluated at $(u^1, v^1)$ and expressed as

$$H = \begin{bmatrix} H_{uu} & H_{uv} \\ H_{vu} & H_{vv} \end{bmatrix} \tag{10}$$

where each submatrix has the form

$$H_{uv} = H_{vu}^T = \begin{bmatrix} \frac{\partial^2 f}{\partial u_1 \partial v_1} & \frac{\partial^2 f}{\partial u_1 \partial v_2} & \cdots & \frac{\partial^2 f}{\partial u_1 \partial v_N} \\ \frac{\partial^2 f}{\partial u_2 \partial v_1} & \frac{\partial^2 f}{\partial u_2 \partial v_2} & \cdots & \frac{\partial^2 f}{\partial u_2 \partial v_N} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial^2 f}{\partial u_N \partial v_1} & \frac{\partial^2 f}{\partial u_N \partial v_2} & \cdots & \frac{\partial^2 f}{\partial u_N \partial v_N} \end{bmatrix}. \tag{11}$$

Using the one-dimensional analogy, the minimum of Eqn. (8) may be computed from the matrix form of Newton's formula, as given by

$$\gamma_1 = \gamma_0 + H^{-1}(\gamma_0) G(\gamma_0). \tag{12}$$

However, this requires that the inverse of $H$ be computed. In many cases of practical interest, only an estimate of the minimum of the quadratic is required; therefore, it is more efficient to use the conjugate gradient method to obtain an estimate of this minimum. Making the substitution $z_u = u - u^1$,

$z_v = v - v^1$, and $D = -G$ into Eqn. (8) and excluding the constant yields a standard quadratic function, denoted by $c$ and written as

$$c = \frac{1}{2}\begin{bmatrix} z_u & z_v \end{bmatrix}\begin{bmatrix} H_{uu} & H_{uv} \\ H_{vu} & H_{vv} \end{bmatrix}\begin{bmatrix} z_u \\ z_v \end{bmatrix} - \begin{bmatrix} z_u & z_v \end{bmatrix}\begin{bmatrix} D_u \\ D_v \end{bmatrix}. \tag{13}$$

Minimizing this expression is the same as solving the linear system

$$\begin{bmatrix} H_{uu} & H_{uv} \\ H_{vu} & H_{vv} \end{bmatrix}\begin{bmatrix} z_u \\ z_v \end{bmatrix} = \begin{bmatrix} D_u \\ D_v \end{bmatrix} \tag{14}$$

where the Hessian matrix $H$ is always symmetric and nonsingular but not always positive definite. Therefore, the conjugate gradient algorithm must be general enough to handle this situation. A suitable algorithm with $L$ restarts and $K << N$ conjugate gradient iterations per restart is given as follows (Ref. 5):

*begin loop for l=1,...,L*

*compute* $D^l$, $H^l$ *and set* $z_u^1 = 0, z_v^1 = 0$

$$r_{u,v}^1 = D_{u,v}^l \tag{15}$$

$$q_u = H_{uu}r_u^1 + H_{uv}r_v^1 \tag{16}$$

$$q_v = H_{vu}r_u^1 + H_{vv}r_v^1 \tag{17}$$

$$\beta_0 = \frac{1}{\langle q_u, q_u\rangle + \langle q_v, q_v\rangle} \tag{18}$$

$$p_{u,v}^1 = \beta_0 q_{u,v} \tag{19}$$

*begin loop for k=1,...,K*

$$q_u = H_{uu}p_u^k + H_{uv}p_v^k \tag{20}$$

$$q_v = H_{vu}p_u^k + H_{vv}p_v^k \tag{21}$$

$$\alpha_k = \frac{1}{\langle q_u, q_u\rangle + \langle q_v, q_v\rangle} \tag{22}$$

$$z_{u,v}^{k+1} = z_{u,v}^k + \alpha_k p_{u,v}^k \tag{23}$$

$$r_{u,v}^{k+1} = r_{u,v}^k - \alpha_k q_{u,v} \tag{24}$$

$$err = \frac{\langle r_u^{k+1}, r_u^{k+1}\rangle + \langle r_v^{k+1}, r_v^{k+1}\rangle}{\langle r_u^k, r_u^k\rangle + \langle r_v^k, r_v^k\rangle} \tag{25}$$

*if err < tolerance, terminate k loop, else*

$$q_u = H_{uu} r_u^{k+1} + H_{uv} r_v^{k+1} \tag{26}$$

$$q_v = H_{vu} r_u^{k+1} + H_{vv} r_v^{k+1} \tag{27}$$

$$\beta_k = \frac{1}{\langle q_u, q_u \rangle + \langle q_v, q_v \rangle} \tag{28}$$

$$p_{u,v}^{k+1} = p_{u,v}^k + \beta_k q_{u,v} \tag{29}$$

*—continue k-loop*

$$u, v^{l+1} = u, v^l + z_{u,v}^{k+1} \tag{30}$$

*continue l loop*

The $l$ loop determines the quadratic approximation to the function $f(u, v)$. The gradient and Hessian are computed at the current estimate and the appropriate conjugate gradient parameters are initialized. The $k$ loop obtains an estimate of the minimum of $c$ by computing an estimate of the solution to the linear system described by Eqn. (14). The dominant computational effort for each $k$ iteration is caused by the matrix vector products, which require $\mathcal{O}(8N^2)$ multiplications. The computation of the Hessian requires $\mathcal{O}(\frac{1}{2}TN^2)$ multiplications, where $T$ is the number of multiplications per matrix element. Since $T$ depends on the function, the computational efficiency of the overall algorithm is function-dependent. An alternate algorithm used by Fong and Birgenheier (Ref. 6) and Press *et al.* (Ref. 7) does not require the computation of the Hessian, but implicitly assumes that it is positive definite. This can lead to an unpredictable breakdown in the algorithm.

# IV. Sum Pattern Synthesis

The goal of the sum pattern synthesis is to maximize the power in the main beam direction while simultaneously minimizing the power in the peak sidelobe. Unfortunately, forcing the sidelobe to a lower level at a particular angular location without any constraint on the other sidelobes will merely shift the peak sidelobe to a new location. Because the peak sidelobe location may change, there is no mathematically elegant way to achieve the goal directly. Rather, the same objective may be achieved indirectly, by minimizing the average of the accumulated peak sidelobe power points. The average is defined to be the average of these peak sidelobes, computed at each restart of the minimization algorithm. If $M_l$ denotes the number of distinct peak sidelobe points at the $l$th restart, then the average power, denoted by $P_a(u,v)$, is defined by

$$P_a(u,v) = \frac{1}{M_l}\sum_{i=1}^{M_l} P_i. \tag{31}$$

The power at the $i$th angular point is defined as $P_i = P(u,v,\theta_i,\phi_i)$, where $i = 0$ denotes the main beam location and $i = 1...M$ denotes peak sidelobe points. In order to minimize the sidelobe power while simultaneously maximizing the main beam power, a function $f(u,v)$ is defined as the ratio of the average peak power to the main beam power and is expressed as

$$f(u,v) = \frac{P_a(u,v)}{P_0(u,v)}. \tag{32}$$

Minimizing this function will achieve the desired goal. A possible one-dimensional scenario is illustrated in Figure 3. The peak sidelobe level is found at the initial start. This sidelobe level is then reduced by a partial minimization of $f(u,v)$. At the first restart the peak sidelobe point is still at the same location, so that peak is reduced again. At the second restart a new peak sidelobe location is found, so this peak and the previous peak are averaged and reduced. At the third restart the peak location has changed

again, so the average is updated and $f(u,v)$ is minimized. At the fourth restart another point is added to the average and $f(u,v)$ is minimized. The pattern has reached the steady-state power distribution.

An inherent characteristic of this method is that all the sidelobes will tend to be brought to the same level. It should be noted that all the sidelobe sample points could have been used a priori at each restart without searching for the peak sidelobe point and the results would be the same. However, using the accumulated average has several advantages. The function $f(u,v)$ changes rapidly when few sidelobe points are present, so that the peak may be brought down fast. As more peak points are added to the average, a self-damping effect takes place and reduces the sensitivity of the pattern to changes in $u$ and $v$, allowing a smooth transition to an optimum. Accumulating the peak points is numerically much faster than starting with all points, since the computation time for evaluating $f(u,v)$ is dependent on $M_l$.

The sidelobe region is defined as the hemisphere formed by the limits $0 \leq \phi \leq 2\pi$ and $0 \leq \theta \leq \pi$ minus the circular aperture of the main beam up to the first null. The density of the sample points should be chosen in such a way that the peak sidelobe may be accurately estimated. A good rule of thumb is that the sample separation distance should be no greater than half the main beamwidth of the design pattern. A uniformly spaced square lattice is sufficient and is shown in Fig. 4. The parameter $k_m$ is the radius of the main beam null of the design pattern, increased slightly to allow the beam to broaden as the sidelobe level is reduced.

The components of $D$ and $H$ required for the conjugate gradient method are given as follows:

$$D_u^m = -\frac{\partial f}{\partial u_m} = \frac{1}{P_0}\left[\frac{P_a}{P_0}\frac{\partial P_0}{\partial u_m} - \frac{\partial P_a}{\partial u_m}\right] \tag{33}$$

$$D_v^m = -\frac{\partial f}{\partial v_m} = \frac{1}{P_0}\left[\frac{P_a}{P_0}\frac{\partial P_0}{\partial v_m} - \frac{\partial P_a}{\partial v_m}\right] \tag{34}$$

$$H_{uu}^{mn} = \frac{\partial^2 f}{\partial u_m \partial u_n} = \frac{1}{P_0}\left[D_u^n\frac{\partial P_0}{\partial u_m} + D_u^m\frac{\partial P_0}{\partial u_n} + \frac{\partial^2 P_a}{\partial u_m \partial u_n} - \frac{P_a}{P_0}\frac{\partial^2 P_0}{\partial u_m \partial u_n}\right] \tag{35}$$

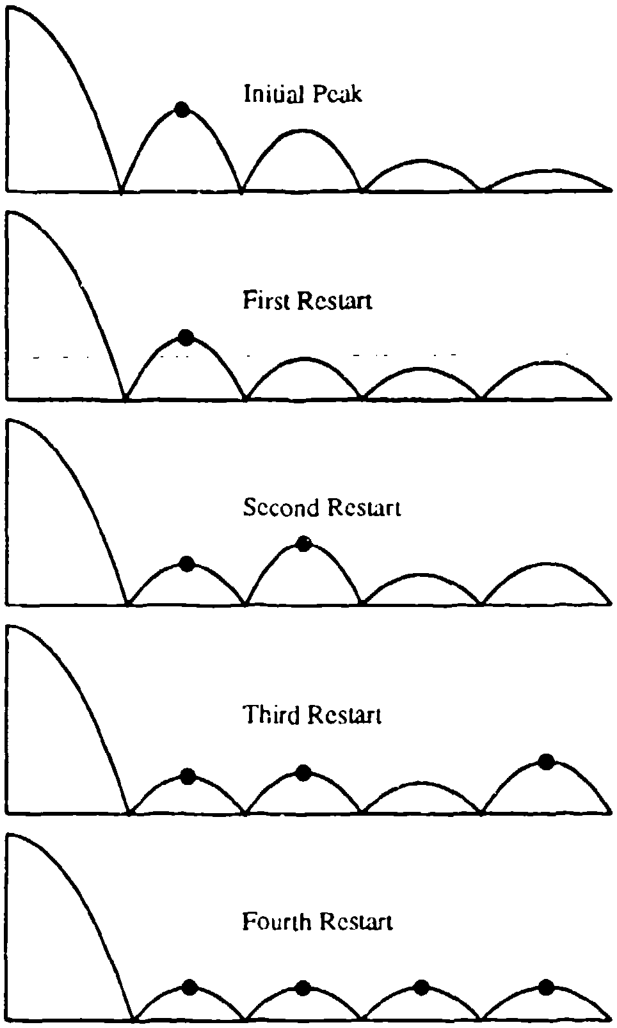



Fig. 3. One-Dimensional Average Sidelobe Accumulation

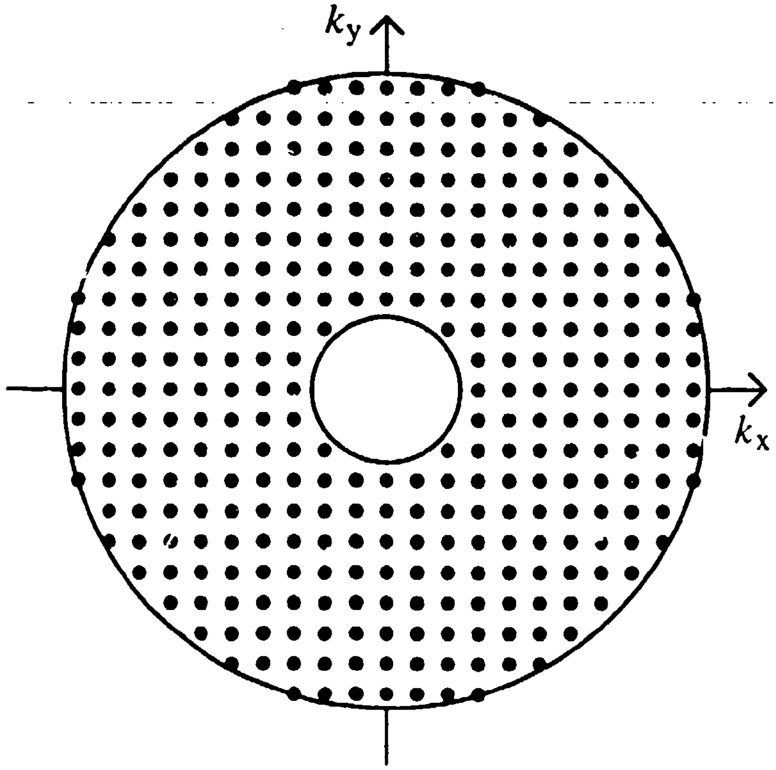



Fig. 4. Sum Pattern Sidelobe Sample Region

$$H_{uv}^{mn} = \frac{\partial^2 f}{\partial u_m \partial v_n} = \frac{1}{P_0}\left[D_v^n \frac{\partial P_0}{\partial u_m} + D_u^m \frac{\partial P_0}{\partial v_n} + \frac{\partial^2 P_a}{\partial u_m \partial v_n} - \frac{P_a}{P_0}\frac{\partial^2 P_0}{\partial u_m \partial v_n}\right] \tag{36}$$

$$H_{vu}^{mn} = H_{uv}^{mn} \tag{37}$$

$$H_{vv}^{mn} = \frac{\partial^2 f}{\partial v_m \partial v_n} = \frac{1}{P_0}\left[D_v^n \frac{\partial P_0}{\partial v_m} + D_v^m \frac{\partial P_0}{\partial v_n} + \frac{\partial^2 P_a}{\partial v_m \partial v_n} - \frac{P_a}{P_0}\frac{\partial^2 P_0}{\partial v_m \partial v_n}\right] \tag{38}$$

The power at the $i$th angle can be computed by introducing the auxiliary functions $\xi_i$ and $\chi_i$ and defining $P_i = \xi_i^2 + \chi_i^2$ such that

$$\xi_i = \sum_{w=1}^{N} [u_w \cos(\psi_{wi}) - v_w \sin(\psi_{wi})] \tag{39}$$

$$\chi_i = \sum_{w=1}^{N} [u_w \sin(\psi_{wi}) + v_w \cos(\psi_{wi})] \tag{40}$$

where $\psi_{wi} = k_{xi} x_w + k_{yi} y_w$. The first-derivatives of the power used in the gradient are given by

$$\frac{\partial P_i}{\partial u_m} = 2\left[\xi_i \cos(\psi_{mi}) + \chi_i \sin(\psi_{mi})\right] \tag{41}$$

$$\frac{\partial P_i}{\partial v_m} = 2\left[\chi_i \cos(\psi_{mi}) - \xi_i \sin(\psi_{mi})\right] \tag{42}$$

and the second derivative terms used in the Hessian are expressed as

$$\frac{\partial^2 P_i}{\partial u_m \partial u_n} = 2\cos(\psi_{mi} - \psi_{ni}) \tag{43}$$

$$\frac{\partial^2 P_i}{\partial u_m \partial v_n} = 2\sin(\psi_{mi} - \psi_{ni}) \tag{44}$$

$$\frac{\partial^2 P_i}{\partial v_m \partial v_n} = \frac{\partial^2 P_i}{\partial u_m \partial u_n}. \tag{45}$$

Note that the computation of the Hessian requires the evaluation of the power at the main beam location and at each peak sidelobe point. Since the power computation requires $\mathcal{O}(N)$ multiplications, the multiplications per matrix element is $T = \mathcal{O}(M_l N)$. Therefore, the Hessian computation dominates the computation of a single $l$ iteration.

# V. Difference Pattern Synthesis

The synthesis procedure for the difference pattern is similar to that used for the sum pattern, except that now only amplitude synthesis is necessary. The appropriate function defined in Eqn. (32) is reduced to one dimension as

$$f(u) = \frac{P_a(u)}{P_0(u)}. \tag{46}$$

The power at the $i$th location is defined as $P_i = \xi_i^2$, where

$$\xi_i = \sum_{w=1}^{N} u_w \sin(\psi_{wi}). \tag{47}$$

The first derivative of the power is computed from

$$\frac{\partial P_i}{\partial u_m} = 2\xi_i \sin(\psi_{mi}) \tag{48}$$

with the second derivative given by

$$\frac{\partial^2 P_i}{\partial u_m \partial u_n} = 2 \sin(\psi_{mi}) \sin(\psi_{ni}). \tag{49}$$

For the configuration shown in Fig. 1, the power pattern is symmetric across the $y-z$ plane. Thus, the $k$-space sample region need only contain points where $k_x > 0$, as shown in Fig. 5.

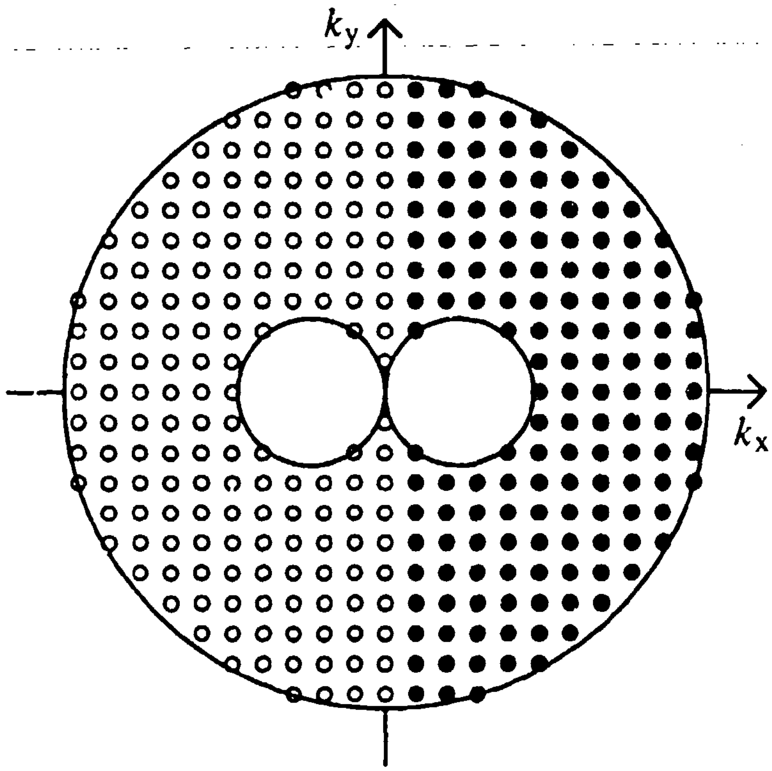



Fig. 5. Difference Pattern Sidelobe Sample Region

## VI. Results and Discussion

A possible element failure scenario is shown in Fig. 6. The hexagonal array has a total of 91 elements spaced $0.6\lambda_0$ apart with 3 element failures. The desired aperture function for the sum pattern is a parabolic with power $N=2$ and an edge taper of -20 dB. This yields a -35 db sidelobe level. The aperture function for the difference pattern is a Bayliss distribution with $\bar{n}=10$ and a -40 dB sidelobe level. All the power patterns are plotted in dB using cylindrical coordinates, with $\theta$ as the radial coordinate in the range $0 \leq \theta \leq 90°$. A reference -32 dB floor is placed on each plot.

The sum power pattern with no element failures is shown in Fig. 7. The peak sidelobe level is -30.9 dB with a peak gain of 20.8 dB. The main beam has azimuthal symmetry with a half-power beamwidth of 12.86°. The synthesized sum pattern of Fig. 8 required 300 restarts, with 5 conjugate gradient iterations per restart, to solve for the 176 required unknowns. The effect of the failures is to increase the sidelobe level to -27.4 dB and reduce the gain to 20.5 dB. The main beam becomes slightly elliptical with a minimum half-power beamwidth of 12.57° and a maximum of 13.00°. The synthesized pattern, shown in Fig. 9, has a peak sidelobe level of -31.3 dB and a gain of 19.5 dB. The main beam also has an elliptical cross section with a minimum half-power beamwidth of 13.71° and a maximum of 16.29°. As expected, the reduction in the sidelobe level produces a corresponding increase in the beamwidth and a drop in gain.

The difference power pattern with no element failures is shown in Fig. 10. The peak sidelobe level is -33.0 dB with a peak gain of 18.4 dB. The angular half-power null width between the peaks is 8.57°. Fig. 11 shows that the effect of the element failures is to increase the sidelobe level to -21.7 dB and drop the gain to 18.0 dB, with the half-power null width remaining about the same. The synthesized difference pattern of Fig. 12 required 100 restarts with 5 conjugate gradient iterations per restart to solve for the 80 unknowns.

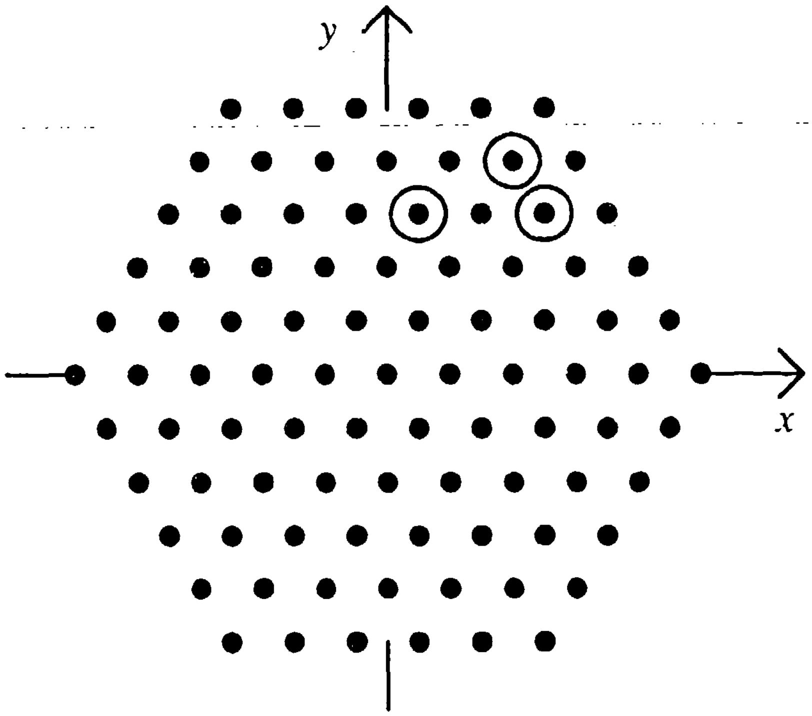



Fig. 6. Hexagonal 91 Element Array with 3 Element Failures

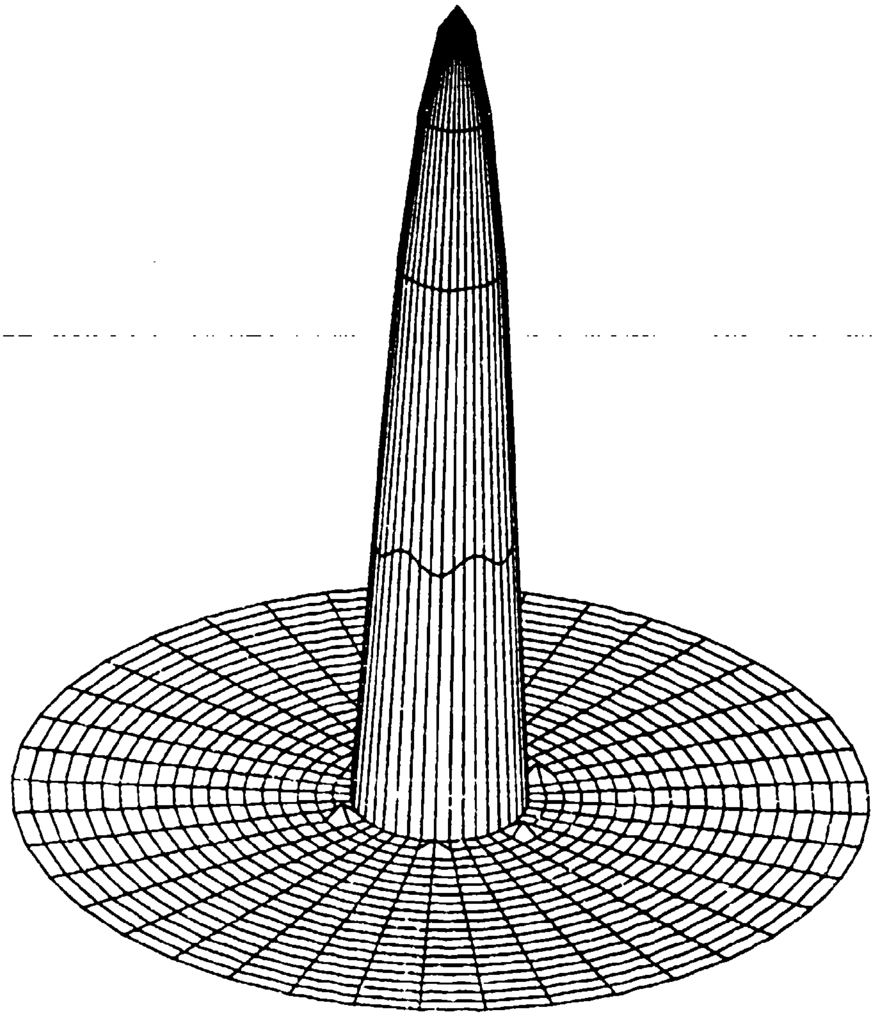


Fig. 7. Sum Power Pattern with No Failures

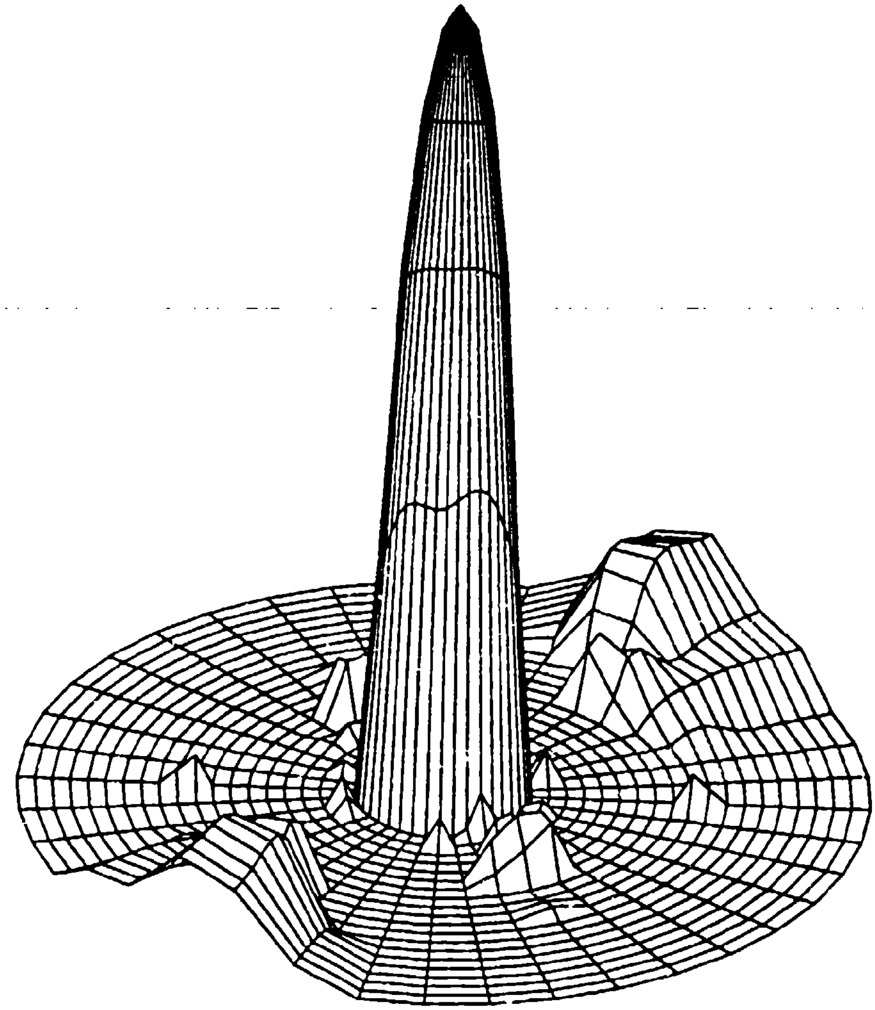


Fig. 8. Sum Power Pattern with 3 Element Failures

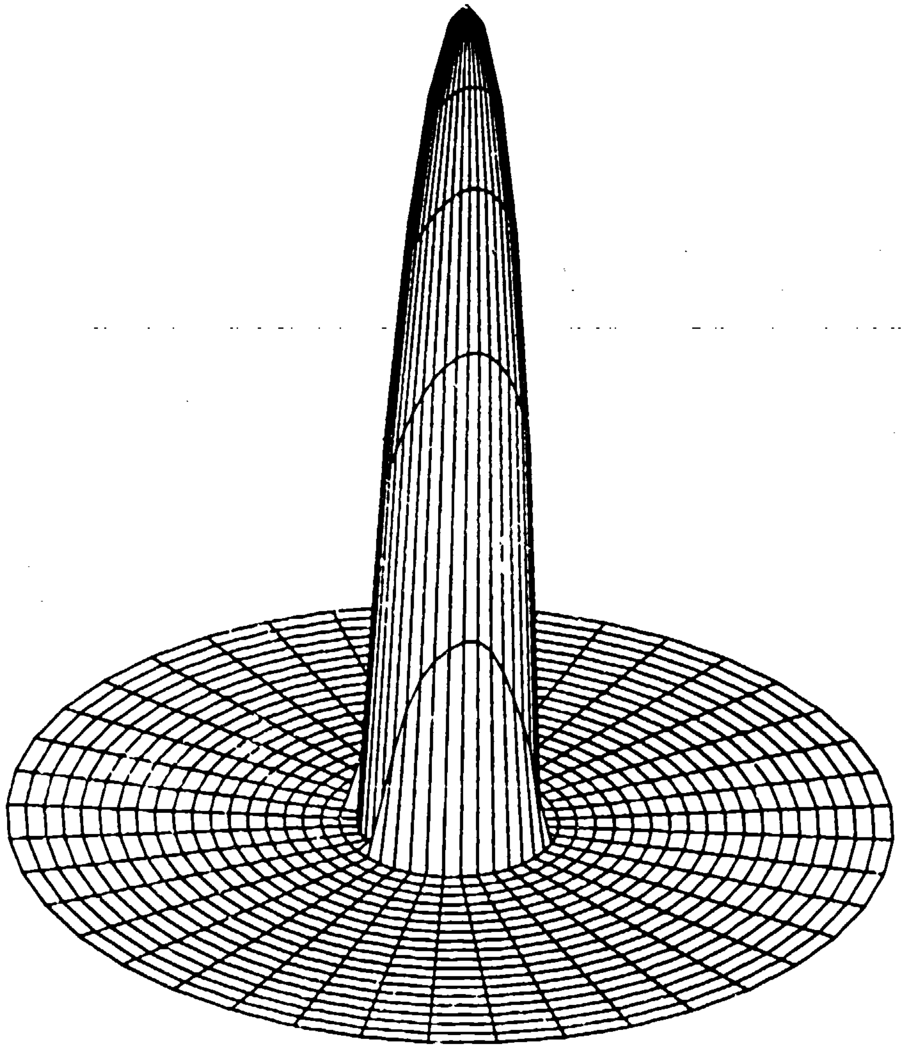


Fig. 9. Synthesized Sum Power Pattern with 3 Element Failures

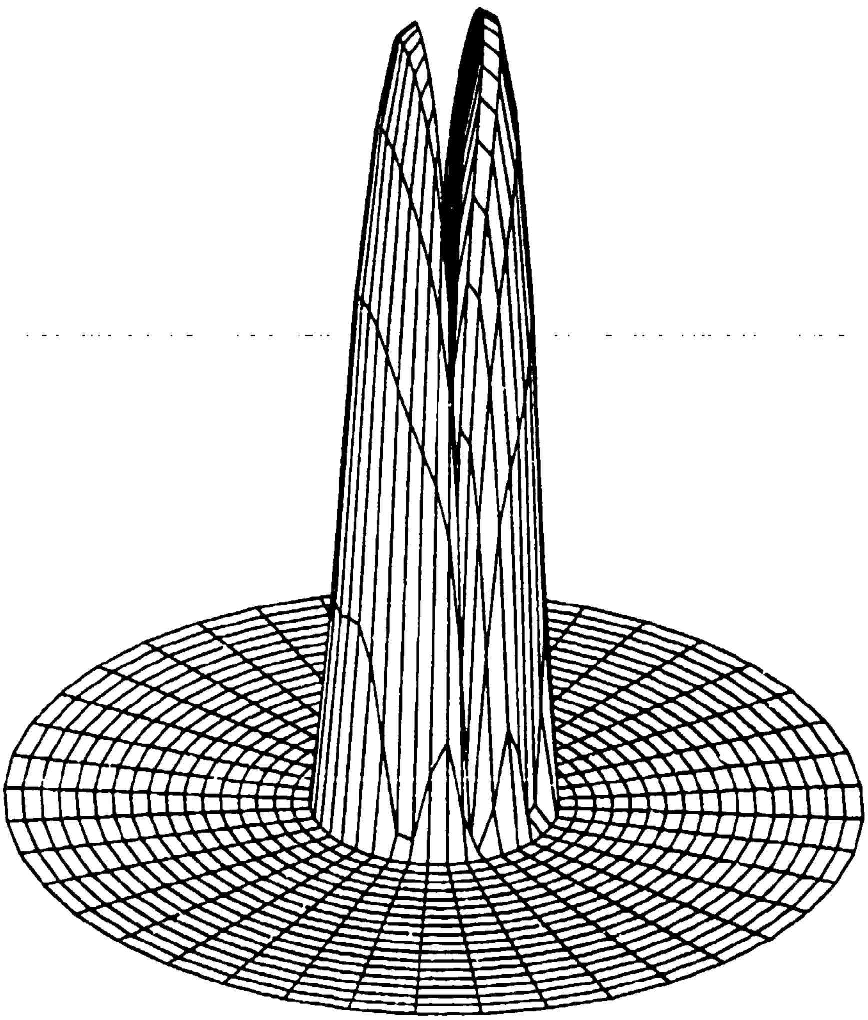


Fig. 10. Difference Power Pattern with No Failures

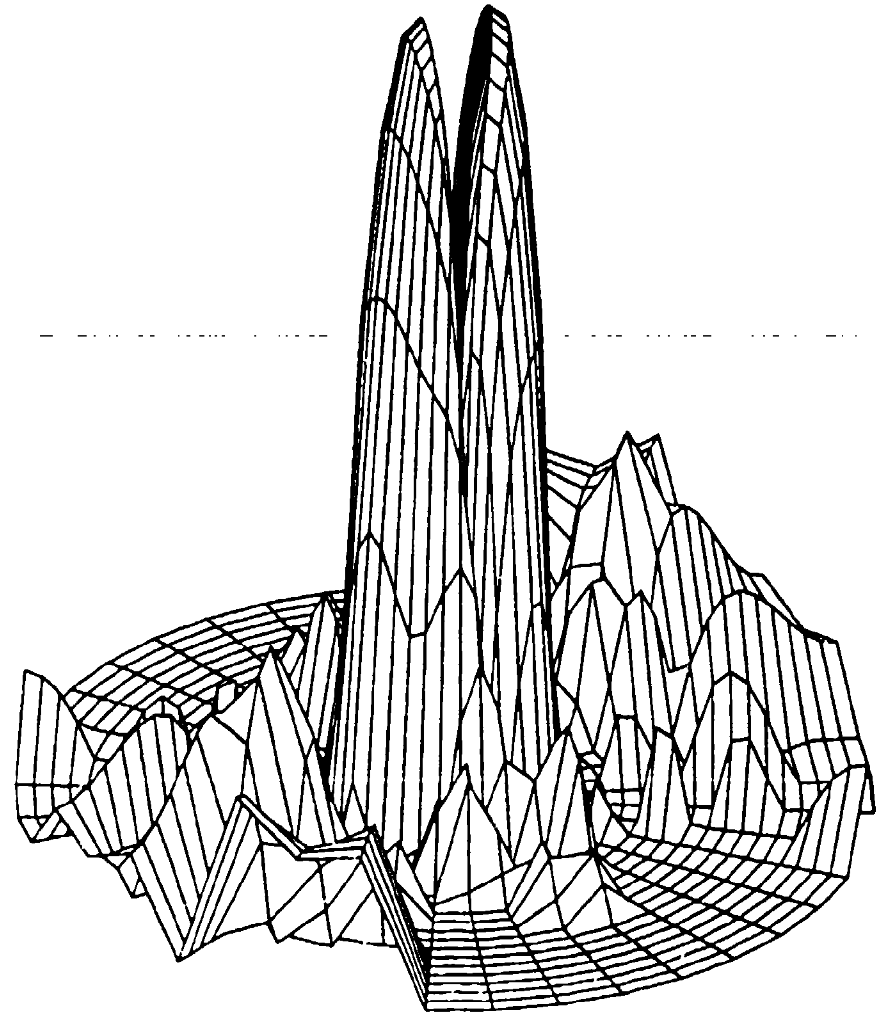


Fig. 11. Difference Power Pattern with 3 Element Failures

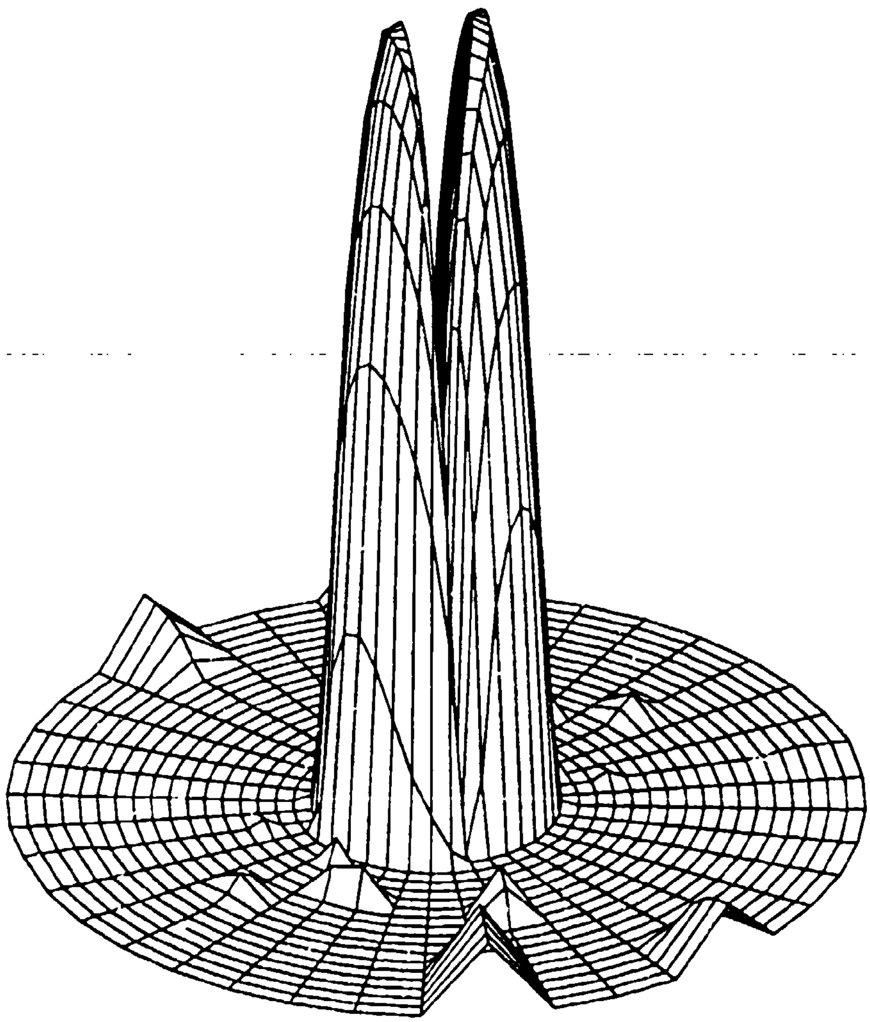


Fig. 12. Synthesized Difference Power Pattern with 3 Element Failures

The peak sidelobe level is -28.7 dB with a gain of 17.1 dB. The sidelobe level of the synthesized pattern cannot be brought down to the level of the design pattern as was the case with the sum pattern, because the degrees of freedom are now about halved. The half-power null width between peaks increased to 9.09°.

The results of this test case and several others, indicate that it is possible to reduce the sidelobe level, of an array with failed elements, down to near the design level. However, the success depends on the nature of the failure. For example, if the central element fails in a sum pattern, the synthesis yields no improvement. This is to be expected since this failure is quite similar to blockage. As expected, elements which fail and have a large relative weight are much more difficult to recover from than elements with a small weight. Also, clustered failures are easier to recover from than the same number of random failures. This is because a clustered failure yields a relatively local increase in the sidelobe region as opposed to the same number of random failures which yields a uniform increase in the sidelobe level. It was interesting to observe that is was easier to recover from an entire row failure than from the same number of random failures. In general, the decrease in the sidelobe level comes with a drop in gain and a slightly broader main beam.

# VII. Conclusion

The performance degradation of arrays with element failures may be partially compensated for by a redistribution of the amplitude and phase over the remaining elements. The extent of this compensation is determined by the number and location of the failed elements. The accumulated averaging scheme, combined with the conjugate gradient algorithm, provides a very stable means to synthesize the new array aperture distribution.

# References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3. M. R. Hestenes and E. Steifel, "Method of Conjugate Gradients for Solving Linear Systems," J. Res. Nat. Bur. Standard., vol. 49, no. 6, pp. 409-436, Dec. 1952.

4. M. R. Hestenes, " Conjugate Direction Methods in Optimization," New York: Springer-Verlag, 1980, pp.135-140.

5. T. J. Peters, " Computation of the Scattering by Planar and Non-Planar Plates Using a Conjugate Gradient FFT Method," Ph.D. dissertation, Radiation Laboratory, University of Michigan, 1988.

6. T. S. Fong and R. A. Birgenheier, "Method of Conjugate Gradients for Antenna Pattern Synthesis," Radio Sci., vol. 6, no. 12, pp. 1123-1130, Dec. 1971.

7. W. A. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, "Conjugate Gradient Methods in Multidimensions," in Numerical Recipes: Cambridge Cambridge University Press, 1988, pp. 301-307.

# Appendix: Fortran 77 Programs

This appendix contains the Fortran 77 programs SUMSYN and DIFSYN as well as all associated subroutines. The program SUMSYN performs the sum pattern synthesis with element failures and the program DIFSYN performs difference pattern synthesis with element failures. The required inputs to each program are listed in the comments found in the source code. These programs were used to generate all the results presented in this report.

```
      PROGRAM SUMSYN
C     ******************************************************************
C     * THIS PROGRAM SYNTHESIZES THE AMPLITUDE AND PHASE DISTRIBUTION  *
C     * NECESSARY TO GENERATE A SUM PATTERN OVER A HEXAGONAL ARRAY OF  *
C     * POINT SOURCES WITH SOME ELEMENT FAILURES.                      *
C     ******************************************************************
C     * TIMOTHY J. PETERS                              LAST UPDATED    *
C     * THE AEROSPACE CORPORATION                      3/1/91          *
C     * 2350 EAST EL SEGUNDO BOULEVARD.                                *
C     * EL SEGUNDO, CA 90245                                           *
C     ******************************************************************
C     * INPUTS:                                                        *
C     *                                                                *
C     * NL  - NUMBER OF LAYERS FORMING THE HEXAGONAL LATTICE           *
C     * NTE - NUMBER OF TOTAL ELEMENTS = 1+3*NL*(NL+1)                 *
C     * NBE - NUMBER OF BAD ELEMENTS                                   *
C     * BAD(1..NBE) - INTEGER ARRAY HOLDING THE NUMBER POSITION OF EACH *
C     *       BAD ELEMENT IN THE ARRAY.                                *
C     * DS  - THE ELEMENT SPACING IN WAVELENGTHS.                      *
C     * NE  - NUMBER OF ACTUAL ELEMENTS = NTE-NBE.                     *
C     * ET  - THE EDGE TAPER OF THE PARABOLIC DISTRIBUTION.            *
C     * N   - THE ORDER OF THE PARABOLIC DISTRIBUTION.                 *
C     * BNF - THE THEORETICAL BEAM WIDTH NULL FACTOR FOR THE CHOSEN    *
C     *       DISTRIBUTION.                                            *
C     * BBF - THE BEAM WIDTH BROADENING FACTOR.  THIS ALLOWS THE BEAM  *
C     *       TO BROADEN BEYOND THE DESIGN VALUE IN ORDER TO REDUCE THE *
C     *       SIDELOBES WHEN ELEMENTS FAIL.                            *
C     * NSP - NUMBER OF SAMPLE POINTS IN THE SIDELOBE REGION.          *
C     * NS  - NUMBER OF RESTARTS.                                      *
C     * NI  - NUMBER OF CONJUGATE GRADIENT ITERATIONS PER RESTART.     *
C     *                                                                *
C     * OUTPUTS:                                                       *
C     *                                                                *
C     * U(NE) - REAL PART OF THE EXCITATION OF EACH ELEMENT.           *
C     * V(NE) - IMAGINARY PART OF THE EXCITATION OF EACH ELEMENT.      *
C     *                                                                *
C     ******************************************************************
      PARAMETER (NL=5,NTE=91,NBE=3,NE=88,NSP=1550)
      REAL*4 U(NE),V(NE),X(NE),Y(NE),SU(NE),SV(NE),GU(NE),GV(NE)
      REAL*4 HUU(NE,NE),HUV(NE,NE),HVU(NE,NE),HVV(NE,NE)
      REAL*4 PU(NE),PV(NE),QU(NE),QV(NE),RU(NE),RV(NE),ZU(NE),ZV(NE)
      REAL*4 KX(NSP),KY(NSP)
      INTEGER IP(NSP),BAD(NE)
      OPEN(UNIT=2,FILE='OUTPUT',STATUS='UNKNOWN')
      OPEN(UNIT=4,FILE='SUM_FIXED_EST',STATUS='UNKNOWN')
C     ******************************************************************
C     * GENERATE THE GEOMETRY.                                         *
C     ******************************************************************
      DS=0.6
```

```
      BAD(1)=35
      BAD(2)=37
      BAD(3)=44
      CALL GEOMET(NL,DS,NE,NBE,BAD,X,Y)
C     ******************************************************************
C     * INITIALIZE THE TAPER DISTRIBUTION.                             *
C     ******************************************************************
      ET=0.1
      N=2
      BNF=1.5
      IFLAG1=0
      IF (IFLAG1 .EQ.-1) THEN
        CALL TAPER(NE,U,V,X,Y,ET,N,NL,DS)
        REWIND 4
        WRITE(4,101) (I,U(I),V(I),I=1,NE)
      ELSE
      END IF
      REWIND 4
      READ(4,101) (I,U(I),V(I),I=1,NE)
C     ******************************************************************
C     * COMPUTE THE GAIN (DIRECTIVITY) OF THE ARRAY.                   *
C     ******************************************************************
      CALL GAIN(NE,U,V,X,Y,GDB)
C     ******************************************************************
C     * GET THE BEAM WIDTH BETWEEN FIRST NULLS.                        *
C     ******************************************************************
      CALL BEAM(NE,U,V,X,Y,BWFN)
C     ******************************************************************
C     * GENERATE THE K-SPACE SAMPLE POINTS.                            *
C     ******************************************************************
      BBF=7.0
      CALL KSPACE(NMP,NL,DS,BNF,BBF,KX,KY)
C     ******************************************************************
C     * COMPUTE POWER IN MAIN BEAM.                                    *
C     ******************************************************************
      CALL POWI(NE,U,V,X,Y,0.0,0.0,PO)
C     ******************************************************************
C     * COMPUTE PEAK SIDELOBE POWER.                                   *
C     ******************************************************************
      NP=0
      CALL PEAKSL(NE,U,V,X,Y,NSP,NMP,KX,KY,IP,NP,PMAX)
C     ******************************************************************
C     * COMPUTE NORMALIZED PEAK SIDELOBE POWER IN DB.                  *
C     ******************************************************************
      PMAXDB=10.0*ALOG10(PMAX/PO)
C     ******************************************************************
C     * PERFORM CONJUGATE GRADIENT STEPS.                              *
C     ******************************************************************
      NS=300
```

```
      NI=3
      CALL CGRAD(NE,NSP,NMP,NS,NI,U,V,X,Y,SU,SV,NP,IP,KX,KY,GU,GV,HUU
     &,HUV,HVU,HVV,PU,PV,QU,QV,RU,RV,ZU,ZV)
      REWIND 4
      WRITE(4,101) (I,U(I),V(I),I=1,NE)
C     *****************************************************************
C     * POWER PATTERN GRAPHICS OUTPUT                                 *
C     *****************************************************************
      REWIND 4
      READ(4,101) (I,U(I),V(I),I=1,NE)
C       ***************************************************************
C       * PRINT THE POWER PATTERN IN CYLINDRICAL COORDINATES.         *
C       ***************************************************************
         CALL POWPAT(NE,U,V,X,Y)
C       ***************************************************************
C       * PRINT THE POWER PATTERN IN SPHERICAL COORDINATES.           *
C       ***************************************************************
         CALL SPOWPAT(NE,U,V,X,Y)
C     *****************************************************************
C     * FORMATS.                                                      *
C     *****************************************************************
 100  FORMAT(I6,1X,F16.6,1X,F16.6)
 101  FORMAT(I6,1X,F16.7,1X,F16.7)
      END
C
      SUBROUTINE GEOMET(NL,S,NE,NBE,BAD,X,Y)
C     *****************************************************************
C     * COMPUTE THE POSITION OF EACH ELEMENT IN A HEXOGONAL ARRAY.    *
C     *****************************************************************
      REAL*4 X(NE),Y(NE)
      INTEGER BAD(NE),IFLAG
      HGT=(SQRT(3.0)/2.0)*S
C     *****************************************************************
C     * CENTER ROW.                                                   *
C     *****************************************************************
      K=0
      L=0
      XMIN=-NL*S
      DO 1 I=0,2*NL
       K=K+1
       L=L+1
       CALL CHECK(L,NE,NBE,BAD,IFLAG)
       IF (IFLAG .EQ. 1) THEN
         K=K-1
       ELSE
         X(K)=XMIN+I*S
         Y(K)=0.0
       END IF
 1    CONTINUE
```

```
C     ********************************************************************
C     * TOP ROWS.                                                        *
C     ********************************************************************
      INUM=2*NL+1
      DO 2 J=1,NL
        INUM=INUM-1
        XMIN=-NL*S+J*S/2.0
        DO 3 I=0,INUM-1
          K=K+1
          L=L+1
          CALL CHECK(L,NE,NBE,BAD,IFLAG)
          IF (IFLAG .EQ. 1) THEN
            K=K-1
          ELSE
            X(K)=XMIN+I*S
            Y(K)=HGT*J
          END IF
 3      CONTINUE
 2    CONTINUE
C     ********************************************************************
C     * BOTTOM ROWS.                                                     *
C     ********************************************************************
      INUM=2*NL+1
      DO 4 J=1,NL
        INUM=INUM-1
        XMIN=-NL*S+J*S/2.0
        DO 5 I=0,INUM-1
          K=K+1
          L=L+1
          CALL CHECK(L,NE,NBE,BAD,IFLAG)
          IF (IFLAG .EQ. 1) THEN
            K=K-1
          ELSE
            X(K)=XMIN+I*S
            Y(K)=-HGT*J
          END IF
 5      CONTINUE
 4    CONTINUE
      RETURN
      END
C
      SUBROUTINE CHECK(L,NE,NBE,BAD,IFLAG)
      INTEGER BAD(NE)
      IFLAG=0
      DO 1 I=1,NBE
         IF (L .EQ. BAD(I)) THEN
            IFLAG=1
         ELSE
         END IF
```

```
1     CONTINUE
      RETURN
      END
C
      SUBROUTINE BEAM(NE,U,V,X,Y,BWFN)
C     ******************************************************************
C     * COMPUTE THE BEAM WIDTH BETWEEN FIRST NULLS.                    *
C     ******************************************************************
      REAL U(NE),V(NE),X(NE),Y(NE),KXI,KYI,KXP,KYP
      RAD=.17453293E-01
      TP=.6283186E+01
      TMIN=0.0
      TMAX=10.0
      P=0.0
      NT=140
      DT=(TMAX-TMIN)/NT
C     ******************************************************************
C     * FIND THE MAXIMUM VALUE.                                        *
C     ******************************************************************
      CALL POWER(NE,U,V,X,Y,0.0,0.0,PEAK)
C     ******************************************************************
C     * RECOMPUTE AND NOW FIND THE POINT WHICH IS AT THE SLL BELOW THE *
C     * PEAK.                                                          *
C     ******************************************************************
      DO 1 K=1,90
        P=P+1.0
        DO 2 I=0,NT
          T=TMIN+I*DT
          TPS=TP*SIN(RAD*T)
          KXI=TPS*COS(RAD*P)
          KYI=TPS*SIN(RAD*P)
          CALL POWI(NE,U,V,X,Y,KXI,KYI,POW)
          IF (POW/PEAK .LE. 0.5) THEN
            WRITE(2,100) T,10.0*ALOG(POW/PEAK)
            GO TO 99
          ELSE
          END IF
 2      CONTINUE
99      CONTINUE
        BWFN=2.0*T
        WRITE(*,*) 'P BWFN ',P,BWFN
1     CONTINUE
100   FORMAT(F10.5,1X,F10.5)
      RETURN
      END
C
      SUBROUTINE KSPACE(NMP,NL,DS,BNF,DBF,KX,KY)
C     ******************************************************************
C     * THIS SUBROUTINE SAMPLES THE KX >0 REGION OF K SPACE.           *
```

```
C     ******************************************************************
      REAL*4 KX(*),KY(*),KC,KR,KXX,KYY,KXMIN,KXMAX,KYMIN,KYMAX
      RAD=.17453293E-01
      PI=.3141593E+01
      TP=.6283185E+01
C     ******************************************************************
C     * COMPUTE THE APPROXIMATE DIAMETER OF THE APERTURE.              *
C     ******************************************************************
      D=2.0*DS*FL
C     ******************************************************************
C     * COMPUTE THE BEAM WIDTH BETWEEN FIRST NULLS OF THE DESIGN       *
C     * PATTERN.                                                       *
C     ******************************************************************
      BWFN=BNF*2.439272/D
C     ******************************************************************
C     * COMPUTE THE SMALLEST BEAM WIDTH BETWEEN FIRST NULLS ALLOWED BY *
C     * THE SYNTHESIS ALGORITHM.                                       *
C     ******************************************************************
      BWFNS=BDF*BWFN
C     ******************************************************************
C     * COMPUTE THE K SPACE RADIUS OF THE ALLOWED MAIN BEAM.           *
C     ******************************************************************
C      KC=TP*0.34
      KC=BWFNS/2.0
C     ******************************************************************
C     * GENERATE THE RECTANGULAR LATTICE OF SAMPLE POINTS.             *
C     ******************************************************************
      KYMIN=-TP
      KYMAX=TP
      KXMIN=-TP
      KXMAX=TP
      NKX=45
      NKY=45
      DKX=(KXMAX-KXMIN)/(NKX-1)
      DKY=(KYMAX-KYMIN)/(NKY-1)
      NMP=0
      DO 1 I=0,NKY
        KYY=KYMIN+I*DKY
        DO 2 J=0,NKX
          KXX=KXMIN+J*DKX
          KR=SQRT(KXX*KXX+KYY*KYY)
          IF ((KR .GT. KC) .AND. (KR .LT. TP)) THEN
            NMP=NMP+1
            KX(NMP)=KXX
            KY(NMP)=KYY
          ELSE
          END IF
 2      CONTINUE
 1    CONTINUE
```

```
      RETURN
      END
C
      SUBROUTINE TAPER(NE,U,V,X,Y,ET,N,NL,S)
      REAL U(NE),V(NE),X(NE),Y(NE)
C    ********************************************************************
C    * COMPUTE APPROXIMATE ARRAY RADIUS.                                *
C    ********************************************************************
     A=S*NL
C    ********************************************************************
C    * COMPUTE AMPLITUDE.                                               *
C    ********************************************************************
      DO 1 K=1,NE
       R=SQRT(X(K)*X(K)+Y(K)*Y(K))
       AMP=ET+(1.0-ET)*((1.0-(R/A)**2)**N)
       U(K)=AMP
       V(K)=0.0
1     CONTINUE
      RETURN
      END
C
      SUBROUTINE CGRAD(NE,NSP,NMP,NS,NI,U,V,X,Y,SU,SV,NP,IP,KX,KY,GU,GV
     &                        ,HUU,HUV,HVU,HVV,PU,PV,QU,QV,RU,RV,ZU,ZV)
C    ********************************************************************
C    * COMPUTE POWER IN A SINGLE DIRECTION.                             *
C    ********************************************************************
      REAL*4 U(NE),V(NE),X(NE),Y(NE),SU(NE),SV(NE),GU(NE),GV(NE)
      REAL*4 HUU(NE,NE),HUV(NE,NE),HVU(NE,NE),HVV(NE,NE)
      REAL*4 PU(NE),PV(NE),QU(NE),QV(NE),RU(NE),RV(NE),ZU(NE),ZV(NE)
      REAL*4 KX(NSP),KY(NSP)
      REAL*4 KXI,KYI
      INTEGER IP(NSP)
C    ********************************************************************
C    * INITIALIZE THE NUMBER OF SIDELOBE SAMPLE POINTS.                 *
C    ********************************************************************
      NP=0
C    ********************************************************************
C    * PERFORM NS RE-STARTS.                                            *
C    ********************************************************************
      DO 1 L=1,NS
C         ***************************************************************
C         * FIND THE PEAK SIDELOBE POWER POINT AND UPDATE THE NUMBER    *
C         * OF MATCH POINTS.                                            *
C         ***************************************************************
           CALL PEAKSL(NE,U,V,X,Y,NSP,NMP,KX,KY,IP,NP,PMAX)
           WRITE(*,160) (IP(I),I=1,NP)
           WRITE(2,160) (IP(I),I=1,NP)
160        FORMAT(10I5)
C         ***************************************************************
```

```
C     * FIND THE POWER IN THE MAIN BEAM.                                   *
C     **********************************************************************
      KXI=0.0
      KYI=0.0
      CALL POWI(NE,U,V,X,Y,KXI,KYI,P0)
C     **********************************************************************
C     * COMPUTE AND STORE AN ARRAY OF THE 1ST DERIVATIVE OF THE            *
C     * POWER IN THE MAIN BEAM DIRECTION WITH RESPECT TO THE MTH           *
C     * VARIABLE.                                                          *
C     ******************************************************************* ***
      KXI=0.0
      KYI=0.0
      DO 2 M=1,NE
        CALL DPOWIU(NE,U,V,X,Y,M,KXI,KYI,DPWIU)
        SU(M)=DPWIU
        CALL DPOWIV(NE,U,V,X,Y,M,KXI,KYI,DPWIV)
        SV(M)=DPWIV
2     CONTINUE
C     *********************************************************************
C     * COMPUTE AND STORE THE AVERAGE SIDELOBE POWER.                      *
C     **********************************************************************
      PA=0.0
      DO 3 I=1,NP
        CALL POWI(NE,U,V,X,Y,KX(IP(I)),KY(IP(I)),POWERI)
        PA=PA+POWERI
3     CONTINUE
      PA=PA/NP
      WRITE(*,200) L,10.0*ALOG10(PA/P0),10.0*ALOG10(PMAX/P0)
      WRITE(2,200) L,10.0*ALOG10(PA/P0),10.0*ALOG10(PMAX/P0)
200   FORMAT(1X,I5,1X,'AVE PEAK SLL',1X,F16.6,1X,
     &                             'PEAK SLL',1X,F16.6)
C     **********************************************************************
C     * COMPUTE THE NEGATIVE OF THE GRADIENT.                              *
C     **********************************************************************
      DO 4 M=1,NE
C          *****************************************************************
C          * COMPUTE THE DERIVATIVE OF THE AVERAGE SIDELOBE POWER.          *
C          ****************************************************************
           DPAMU=0.0
           DPAMV=0.0
           DO 5 I=1,NP
             CALL DPOWIU(NE,U,V,X,Y,M,KX(IP(I)),KY(IP(I)),DPWIU)
             DPAMU=DPAMU+DPWIU
             CALL DPOWIV(NE,U,V,X,Y,M,KX(IP(I)),KY(IP(I)),DPWIV)
             DPAMV=DPAMV+DPWIV
5          CONTINUE
           DPAMU=DPAMU/NP
           DPAMV=DPAMV/NP
C          ****************************************************************
```

```
C           * COMPUTE THE NEGATIVE GRADIENT.                              *
C           ***************************************************************
             GU(M)=(PA*SU(M)/PO-DPAMU)/PO
             GV(M)=(PA*SV(M)/PO-DPAMV)/PO
 4        CONTINUE
C        *********************************************************************
C        * COMPUTE THE SYMMETRIC HESSIAN SUB MATRICES HUU AND HVV          *
C        *********************************************************************
          DO 6 M=1,NE
            DO 7 N=M,NE
              KXI=0.0
              KYI=0.0
              CALL DDPIUU(NE,X,Y,M,N,KXI,KYI,DDPUU)
              DDPOUU=DDPUU
              CALL DDPIVV(NE,X,Y,M,N,KXI,KYI,DDPVV)
              DDPOVV=DDPVV
              DDPAUU=0.0
              DDPAVV=0.0
              DO 8 I=1,NP
                CALL DDPIUU(NE,X,Y,M,N,KX(IP(I)),KY(IP(I)),DDPUU)
                DDPAUU=DDPAUU+DDPUU
                CALL DDPIVV(NE,X,Y,M,N,KX(IP(I)),KY(IP(I)),DDPVV)
                DDPAVV=DDPAVV+DDPVV
 8            CONTINUE
              DDPAUU=DDPAUU/NP
              DDPAVV=DDPAVV/NP
              HUU(M,N)=(GU(N)*SU(M)+GU(M)*SU(N)+DDPAUU-PA*DDPOUU/PO)/PO
              HUU(N,M)=HUU(M,N)
              HVV(M,N)=(GV(N)*SV(M)+GV(M)*SV(N)+DDPAVV-PA*DDPOVV/PO)/PO
              HVV(N,M)=HVV(M,N)
 7          CONTINUE
 6        CONTINUE
C        *********************************************************************
C        * COMPUTE THE ASYMMETRIC HESSIAN SUB MATRICE HUV.                 *
C        *********************************************************************
          DO 9 M=1,NE
            DO 10 N=1,NE
              KXI=0.0
              KYI=0.0
              CALL DDPIUV(NE,X,Y,M,N,KXI,KYI,DDPUV)
              DDPOUV=DDPUV
              DDPAUV=0.0
              DO 11 I=1,NP
                CALL DDPIUV(NE,X,Y,M,N,KX(IP(I)),KY(IP(I)),DDPUV)
                DDPAUV=DDPAUV+DDPUV
 11           CONTINUE
              DDPAUV=DDPAUV/NP
              HUV(M,N)=(GV(N)*SU(M)+GU(M)*SV(N)+DDPAUV-PA*DDPOUV/PO)/PO
              HVU(N,M)=HUV(M,N)
```

```
   10     CONTINUE
    9    CONTINUE
C        ****************************************************************
C        * START THE CONJUGATE GRADIENT ALGORITHM.                      *
C        ****************************************************************
C        ****************************************************************
C        * INITIALIZE THE RESIDUAL.                                     *
C        ****************************************************************
         DO 12 M=1,NE
           RU(M)=GU(M)
           RV(M)=GV(M)
           ZU(M)=0.0
           ZV(M)=0.0
   12    CONTINUE
C        ****************************************************************
C        * INITIALIZE SEARCH VECTOR.                                    *
C        ****************************************************************
         CALL MATVEC(NE,HUU,RU,HUV,RV,QU)
         CALL MATVEC(NE,HVU,RU,HVV,RV,QV)
         CALL NORM22(NE,QU,QUN)
         CALL NORM22(NE,QV,QVN)
         BEO=1.0/(QUN+QVN)
         DO 13 I=1,NE
           PU(I)=BEO*QU(I)
           PV(I)=BEO*QV(I)
   13    CONTINUE
C        ****************************************************************
C        * PERFORM CONJUGATE GRADIENT ITERATIONS.                       *
C        ****************************************************************
         DO 14 K=1,NI
C        ****************************************************************
C        * UPDATE AMPLITUDE VECTOR AND RESIDUAL.                        *
C        ****************************************************************
         CALL MATVEC(NE,HUU,PU,HUV,PV,QU)
         CALL MATVEC(NE,HVU,PU,HVV,PV,QV)
         CALL NORM22(NE,QU,QUN)
         CALL NORM22(NE,QV,QVN)
         AK=1.0/(QUN+QVN)
         DO 15 I=1,NE
           ZU(I)=ZU(I)+AK*PU(I)
           ZV(I)=ZV(I)+AK*PV(I)
           RU(I)=RU(I)-AK*QU(I)
           RV(I)=RV(I)-AK*QV(I)
     15  CONTINUE
         CALL NORM22(NE,RU,RUNM)
         CALL NORM22(NE,RV,RVNM)
         ERR=SQRT(RUNM+RVNM)
         WRITE(*,500) L,K,ERR
         WRITE(2,500) L,K,ERR
```

```
500       FORMAT(3X,'RESIDUAL',1X,I3,1X,I3,1X,E15.8)
C        **************************************************************
C        * UPDATE SEARCH VECTOR.                                      *
C        **************************************************************
          CALL MATVEC(NE,HUU,RU,HUV,RV,QU)
          CALL MATVEC(NE,HVU,RU,HVV,RV,QV)
          CALL NORM22(NE,QU,QUN)
          CALL NORM22(NE,QV,QVN)
          BEK=1.0/(QUN+QVN)
          DO 16 I=1,NE
            PU(I)=PU(I)+BEK*QU(I)
            PV(I)=PV(I)+BEK*QV(I)
  16      CONTINUE
14        CONTINUE
C        **************************************************************
C        * UPDATE AMPLITUDE VECTOR.                                   *
C        **************************************************************
          DO 17 I=1,NE
            U(I)=U(I)+ZU(I)
            V(I)=V(I)+ZV(I)
  17      CONTINUE
          CALL NORMAL(NE,U,V)
C        **************************************************************
C        * WRITE OUT THE NEW ESTIMATE TO A FILE.                      *
C        **************************************************************
        REWIND 4
        WRITE(4,101) (I,U(I),V(I),I=1,NE)
101   FORMAT(I5,1X,F15.7,1X,F15.7)
      IF (NP .EQ. NMP) THEN
        GO TO 99
      ELSE
      END IF
1     CONTINUE
99    CONTINUE
      RETURN
      END
C
      SUBROUTINE PEAKSL(NE,U,V,X,Y,NSP,NMP,KX,KY,IP,NP,PMAX)
C    *****************************************************************
C    * THIS SUBROUTINE FINDS THE PEAK SIDELOBE POWER LEVEL COORDINATE *
C    * POINT (KX(LMAX),KY(LMAX)).  IF THE POINT IS NEW THEN IT IS     *
C    * ACCUMULATED INTO THE POINTER ARRAY IP. THE NUMBER OF POINTS IN *
C    * THE INTEGER POINTER ARRAY IP IS NP SUCH THAT IP(I),I=1...NP.   *
C    *****************************************************************
      REAL U(NE),V(NE),X(NE),Y(NE),KX(NSP),KY(NSP)
      INTEGER IP(NSP)
C        **************************************************************
C        * COMPUTE THE POWER AT EACH POINT AND FIND THE MAX.          *
C        **************************************************************
```

```
            PMAX=-1000.0
            LMAX=-1
            DO 1 I=1,NMP
              CALL POWI(NE,U,V,X,Y,KX(I),KY(I),PWI)
              IF (PWI .GT. PMAX) THEN
                LMAX=I
                PMAX=PWI
              ELSE
              END IF
1           CONTINUE
C           ***********************************************************
C           * COMPARE LMAX TO ALL THE PREVIOUS POINTERS IP(I),I=1...NP.  *
C           * IF LMAX MATCHES ONE OF THE PREVIOUS POINTERS THEN THE      *
C           * POINTER ARRAY NEED NOT BE UPDATED (IFLAG=1).  IF LMAX DOES *
C           * NOT MATCH A PREVIOUS POINTER VALUE (IFLAG=0) THEN IT       *
C           ***********************************************************
            IFLAG=0
            DO 2 I=1,NP
              IF (IP(I) .EQ. LMAX) THEN
                IFLAG=1
              ELSE
              END IF
2           CONTINUE
            IF (IFLAG .EQ. 0) THEN
              NP=NP+1
              IP(NP)=LMAX
            ELSE
            END IF
      RETURN
      END
C
      SUBROUTINE POWI(NE,U,V,X,Y,KXI,KYI,POWERI)
C     ****************************************************************
C     * COMPUTE POWER IN A SINGLE DIRECTION.                         *
C     ****************************************************************
      REAL U(NE),V(NE),X(NE),Y(NE),KXI,KYI
      SUM1=0.0
      SUM2=0.0
      DO 1 K=1,NE
       PSI=KXI*X(K)+KYI*Y(K)
       CP=COS(PSI)
       SP=SIN(PSI)
       SUM1=SUM1+U(K)*CP-V(K)*SP
       SUM2=SUM2+U(K)*SP+V(K)*CP
1     CONTINUE
      TI=SUM1*SUM1
      VI=SUM2*SUM2
      POWERI=TI+VI
      RETURN
```

```
      END
C
      SUBROUTINE DPOWIU(NE,U,V,X,Y,M,KXI,KYI,DPWIU)
C    ******************************************************************
C    * COMPUTE 1ST DERIVATIVE OF POWER WITH RESPECT TO THE MTH VARIABLE*
C    * IN THE ITH DIRECTION.                                           *
C    ******************************************************************
      REAL U(NE),V(NE),X(NE),Y(NE),KXI,KYI
C    ******************************************************************
C    * COMPUTE SUMS.                                                   *
C    ******************************************************************
      SUM1=0.0
      SUM2=0.0
      DO 1 K=1,NE
       PSI=KXI*X(K)+KYI*Y(K)
       CP=COS(PSI)
       SP=SIN(PSI)
       SUM1=SUM1+U(K)*CP-V(K)*SP
       SUM2=SUM2+U(K)*SP+V(K)*CP
1     CONTINUE
C    ******************************************************************
C    * COMPUTE 1ST DERIVATIVE OF POWER.                                *
C    ******************************************************************
      PSIM=KXI*X(M)+KYI*Y(M)
      DPWIU=2.0*(COS(PSIM)*SUM1+SIN(PSIM)*SUM2)
      RETURN
      END
C
      SUBROUTINE DPOWIV(NE,U,V,X,Y,M,KXI,KYI,DPWIV)
C    ******************************************************************
C    * COMPUTE 1ST DERIVATIVE OF POWER WITH RESPECT TO THE MTH VARIABLE*
C    * IN THE ITH DIRECTION.                                           *
C    ******************************************************************
      REAL U(NE),V(NE),X(NE),Y(NE),KXI,KYI
C    ******************************************************************
C    * COMPUTE SUMS.                                                   *
C    ******************************************************************
      SUM1=0.0
      SUM2=0.0
      DO 1 K=1,NE
       PSI=KXI*X(K)+KYI*Y(K)
       CP=COS(PSI)
       SP=SIN(PSI)
       SUM1=SUM1+U(K)*CP-V(K)*SP
       SUM2=SUM2+U(K)*SP+V(K)*CP
1     CONTINUE
C    ******************************************************************
C    * COMPUTE 1ST DERIVATIVE OF POWER.                                *
C    ******************************************************************
```

```
      PSIM=KXI*X(M)+KYI*Y(M)
      DPWIV=2.0*(COS(PSIM)*SUM2-SIN(PSIM)*SUM1)
      RETURN
      END
C
      SUBROUTINE DDPIUU(NE,X,Y,M,N,KXI,KYI,DDPUU)
C     *****************************************************************
C     * COMPUTE 2ND DERIVATIVE OF POWER WITH RESPECT TO THE M AND NTH *
C     * VARIABLES IN THE ITH DIRECTION.                               *
C     *****************************************************************
      REAL*4 X(NE),Y(NE),KXI,KYI
C     *****************************************************************
C     * COMPUTE DIFFERENCE OF PSI FUNCTIONS.                          *
C     *****************************************************************
      PSIM=KXI*X(M)+KYI*Y(M)
      PSIN=KXI*X(N)+KYI*Y(N)
      DIF=PSIM-PSIN
C     *****************************************************************
C     * COMPUTE 2ND DERIVATIVE OF POWER.                              *
C     *****************************************************************
      DDPUU=2.0*COS(DIF)
      RETURN
      END
C
      SUBROUTINE DDPIUV(NE,X,Y,M,N,KXI,KYI,DDPUV)
C     *****************************************************************
C     * COMPUTE 2ND DERIVATIVE OF POWER WITH RESPECT TO THE M AND NTH *
C     * VARIABLES IN THE ITH DIRECTION.                               *
C     *****************************************************************
      REAL*4 X(NE),Y(NE),KXI,KYI
C     *****************************************************************
C     * COMPUTE DIFFERENCE OF PSI FUNCTIONS.                          *
C     *****************************************************************
      PSIM=KXI*X(M)+KYI*Y(M)
      PSIN=KXI*X(N)+KYI*Y(N)
      DIF=PSIM-PSIN
C     *****************************************************************
C     * COMPUTE 2ND DERIVATIVE OF POWER.                              *
C     *****************************************************************
      DDPUV=2.0*SIN(DIF)
      RETURN
      END
C
      SUBROUTINE DDPIVV(NE,X,Y,M,N,KXI,KYI,DDPVV)
C     *****************************************************************
C     * COMPUTE 2ND DERIVATIVE OF POWER WITH RESPECT TO THE M AND NTH *
C     * VARIABLES IN THE ITH DIRECTION.                               *
C     *****************************************************************
      REAL*4 X(NE),Y(NE),KXI,KYI
```

```
C     ******************************************************************
C     * COMPUTE DIFFERENCE OF PSI FUNCTIONS.                           *
C     ******************************************************************
      PSIM=KXI*X(M)+KYI*Y(M)
      PSIN=KXI*X(N)+KYI*Y(N)
      DIF=PSIM-PSIN
C     ******************************************************************
C     * COMPUTE 2ND DERIVATIVE OF POWER.                               *
C     ******************************************************************
      DDPVV=2.0*COS(DIF)
      RETURN
      END
C
      SUBROUTINE NORM22(N,A,AN)
C     ******************************************************************
C     * THIS SUBROUTINE COMPUTES THE EUCLIDIAN NORM SQUARED OF A.      *
C     ******************************************************************
      REAL*4 A(N)
      AN=0.0
      DO 1 I=1,N
       AN=AN+A(I)*A(I)
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE MATVEC(N,A,B,C,D,E)
C     ******************************************************************
C     * THIS SUBROUTINE COMPUTES THE MATRIX PRODUCT E=AB+CD.           *
C     ******************************************************************
      REAL*4 A(N,N),B(N),C(N,N),D(N),E(N)
      DO 1 I=1,N
       SUM=0.0
       DO 2 J=1,N
        SUM=SUM+A(I,J)*B(J)+C(I,J)*D(J)
    2  CONTINUE
       E(I)=SUM
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE NORMAL(N,A,B)
C     ******************************************************************
C     * THIS SUBROUTINE NORMALIZES A VECTOR OF LENGTH N.               *
C     ******************************************************************
      REAL*4 A(N),B(N)
      VMAX=SQRT(A(1)*A(1)+B(1)*B(1))
      DO 1 I=1,N
        V=SQRT(A(I)*A(I)+B(I)*B(I))
        IF (V .GT. VMAX) THEN
```

```
            VMAX=V
          ELSE
          END IF
    1   CONTINUE
        DO 2 I=1,N
         A(I)=A(I)/ABS(VMAX)
         B(I)=B(I)/ABS(VMAX)
    2   CONTINUE
        RETURN
        END
C
        SUBROUTINE GAIN(NE,U,V,X,Y,GDB)
C       **************************************************************
C       * COMPUTE POWER IN A SINGLE DIRECTION.                       *
C       **************************************************************
        REAL U(NE),V(NE),X(NE),Y(NE)
        RAD=.17453293E-01
        PI=.3141593E+01
        TP=.6283186E+01
C       **************************************************************
C       * GRAPH INPUTS                                               *
C       **************************************************************
        NP=41
        NR=31
        PMIN=0.0
        PMAX=360.0
        TMIN=0.0
        TMAX=90.0
        DP=(PMAX-PMIN)/(NP-1)
        DR=(TMAX-TMIN)/(NR-1)
C            *********************************************************
C            * COMPUTE PEAK VALUE.                                   *
C            *********************************************************
             CALL POWER(NE,U,V,X,Y,0.0,0.0,PEAK)
C            *********************************************************
C            * COMPUTE THE SOLID ANGLE.                              *
C            *********************************************************
             SUM=0.0
             DO 1 I=0,NR-2
               T=TMIN+DR/2.0+I*DR
               ST=SIN(RAD*T)
               DO 2 J=0,NP-2
                 P=PMIN+DP/2.0+J*DP
                 CALL POWER(NE,U,V,X,Y,T,P,POW)
                 SUM=SUM+ST*POW
    2          CONTINUE
    1        CONTINUE
             TEST=RAD*RAD*DR*DP*SUM
             SOLID=RAD*RAD*DR*DP*SUM/PEAK
```

```
          GDB=10.0*ALOG10(TP/SOLID)
      RETURN
      END
C
      SUBROUTINE POWPAT(NE,U,V,X,Y)
C    *****************************************************************
C    * COMPUTE POWER IN A SINGLE DIRECTION.                          *
C    *****************************************************************
      REAL U(NE),V(NE),X(NE),Y(NE)
      OPEN(UNIT=7,FILE='DATA')
      RAD=.17453293E-01
      PI=.3141593E+01
      TP=.6283185E+01
C    *****************************************************************
C    * GRAPH INPUTS                                                  *
C    *****************************************************************
      NP=55
      NR=30
      NS=0
      PMIN=0.0
      PMAX=360.0
      TMIN=0.0
      TMAX=90.0
      DP=(PMAX-PMIN)/(NP-1)
      DR=(TMAX-TMIN)/(NR-1)
      AX=150.0
      AY=AX
      AZ=680.0
      P0=55.0
      Q0=60.0
      XA=-2.0
      XB=2.0
      YA=-2.0
      YB=2.0
      ZA=-2.0
      ZB=2.3
      AT=.3
      WRITE(7,*) NP,NR,NS,AX,AY,AZ
      WRITE(7,*) XA,XB,YA,YB,ZA,ZB
      WRITE(7,*) AT,P0,Q0
C    *****************************************************************
C    * PHI VALUES.                                                   *
C    *****************************************************************
      DO 1 I=0,NP-1
       P=PMIN+I*DP
       WRITE(7,*) RAD*P
 1    CONTINUE
C    *****************************************************************
C    * RADIAL (THETA) VALUES.                                        *
```

```
C     ******************************************************************
      DO 2 I=0,NR-1
       T=TMIN+I*DR
       WRITE(7,*) RAD*T
 2    CONTINUE
C     ******************************************************************
C     * FUNCTION VALUES.                                               *
C     ******************************************************************
C          *************************************************************
C          * COMPUTE PEAK VALUE.                                       *
C          *************************************************************
           CALL POWER(NE,U,V,X,Y,0.0,0.0,PEAK)
C          *************************************************************
C          * COMPUTE EACH VALUE.                                       *
C          *************************************************************
           DO 3 I=0,NR-1
             T=TMIN+I*DR
             DO 4 J=0,NP-1
               P=PMIN+J*DP
               CALL POWER(NE,U,V,X,Y,T,P,POW)
               WRITE(7,*) POW/PEAK
C4            CONTINUE
C3         CONTINUE
      RETURN
      END
C
      SUBROUTINE SPOWPAT(NE,U,V,X,Y)
C     ******************************************************************
C     * COMPUTE POWER IN A SINGLE DIRECTION.                           *
C     ******************************************************************
      REAL U(NE),V(NE),X(NE),Y(NE)
      OPEN(UNIT=8,FILE='SDATA')
      RAD=.17453293E-01
      PI=.3141593E+01
      TP=.6283185E+01
C     ******************************************************************
C     * GRAPH INPUTS                                                   *
C     ******************************************************************
      NP=115
      NR=63
      NS=0
      PMIN=0.0
      PMAX=360.0
      TMIN=0.0
      TMAX=90.0
      DP=(PMAX-PMIN)/(NP-1)
      DR=(TMAX-TMIN)/(NR-1)
      AX=600.0
      AY=AX
```

```
      AZ=AX
      P0=25.0
      Q0=60.0
      XA=-2.0
      XB=2.0
      YA=-2.0
      YB=2.0
      ZA=-2.0
      ZB=2.3
      AT=.3
      IT=4
      WRITE(8,*) NP,NR,NS,IT,AX,AY,AZ
      WRITE(8,*) XA,XB,YA,YB,ZA,ZB
      WRITE(8,*) AT,P0,Q0
C     ****************************************************************
C     * PHI VALUES.                                                  *
C     ****************************************************************
      DO 1 I=0,NP-1
       P=PMIN+I*DP
       WRITE(8,*) RAD*P
 1    CONTINUE
C     ****************************************************************
C     * RADIAL (THETA) VALUES.                                       *
C     ****************************************************************
      DO 2 I=0,NR-1
       T=TMIN+I*DR
       WRITE(8,*) RAD*T
 2    CONTINUE
C     ****************************************************************
C     * FUNCTION VALUES.                                             *
C     ****************************************************************
C          ***********************************************************
C          * COMPUTE PEAK VALUE.                                     *
C          ***********************************************************
           CALL POWER(NE,U,V,X,Y,0.0,0.0,PEAK)
C          ***********************************************************
C          * COMPUTE EACH VALUE.                                     *
C          ***********************************************************
           DO 3 I=0,NR-1
             T=TMIN+I*DR
             DO 4 J=0,NP-1
               P=PMIN+J*DP
               CALL POWER(NE,U,V,X,Y,T,P,POW)
               WRITE(8,*) POW/PEAK
 4           CONTINUE
 3         CONTINUE
      RETURN
      END
C
```

```
      SUBROUTINE POWER(NE,U,V,X,Y,T,P,POW)
C    ******************************************************************
C    * COMPUTE POWER IN A SINGLE DIRECTION GIVEN A THETA AND PHI.     *
C    ******************************************************************
      REAL U(NE),V(NE),X(NE),Y(NE),KXI,KYI
      RAD=.17453293E-01
      TP=.6283166E+01
      TPS=TP*SIN(RAD*T)
      RP=RAD*P
      KXI=TPS*COS(RP)
      KYI=TPS*SIN(RP)
      SUM1=0.0
      SUM2=0.0
      DO 1 K=1,NE
       PSI=KXI*X(K)+KYI*Y(K)
       CP=COS(PSI)
       SP=SIN(PSI)
       SUM1=SUM1+U(K)*CP-V(K)*SP
       SUM2=SUM2+U(K)*SP+V(K)*CP
    1 CONTINUE
      TI=SUM1*SUM1
      VI=SUM2*SUM2
      POW=TI+VI
      RETURN
      END
```

```
      PROGRAM DIFSYN
C     ******************************************************************
C     * THIS PROGRAM SYNTHESIZES THE AMPLITUDE AND PHASE DISTRIBUTION  *
C     * NECESSARY TO GENERATE A DIFFERENCE PATTERN OVER A HEXAGONAL    *
C     * ARRAY OF POINT SOURCES WITH SOME ELEMENT FAILURES.             *
C     ******************************************************************
C     * TIMOTHY J. PETERS                             'ST UPDATED      *
C     * THE AEROSPACE CORPORATION                       ./91           *
C     * 2350 EAST EL SEGUNDO BOULEVARD.                                *
C     * EL SEGUNDO, CA 90245                                           *
C     ******************************************************************
C     * INPUTS:                                                        *
C     *                                                                *
C     * NL  - NUMBER OF LAYERS FORMING THE HEXAGONAL LATTICE           *
C     * NTE - NUMBER OF TOTAL ELEMENTS =(1+3*NL*(NL+1)-NL)/2 FOR NL ODD *
C     *                                =(3*NL*(NL+1)-NL)/2 FOR NL EVEN  *
C     * NBE - NUMBER OF BAD ELEMENTS                                   *
C     * BAD(1..NBE) - INTEGER ARRAY HOLDING THE NUMBER POSITION OF EACH *
C     *       BAD ELEMENT IN THE ARRAY.                                *
C     * DS  - THE ELEMENT SPACING IN WAVELENGTHS.                      *
C     * NE  - NUMBER OF ACTUAL ELEMENTS = NTE-NBE.                     *
C     * SLL - SIDELOBE LEVEL OF DESIGN BAYLISS DISTRIBUTION.           *
C     * NBAR- PARAMETER FOR BAYLISS DISTRIBUTION.                      *
C     * BBF - THE BEAM WIDTH BROADENING FACTOR.  THIS ALLOWS THE BEAM  *
C     *       TO BROADEN BEYOND THE DESIGN VALUE IN ORDER TO REDUCE THE *
C     *       SIDELOBES WHEN ELEMENTS FAIL.                            *
C     * NSP - NUMBER OF SAMPLE POINTS IN THE SIDELOBE REGION.          *
C     * NS  - NUMBER OF RESTARTS.                                      *
C     * NI  - NUMBER OF CONJUGATE GRADIENT ITERATIONS PER RESTART.     *
C     *                                                                *
C     * OUTPUTS:                                                       *
C     *                                                                *
C     * U(NE) - REAL PART OF THE EXCITATION OF EACH ELEMENT.           *
C     * V(NE) - IMAGINARY PART OF THE EXCITATION OF EACH ELEMENT.      *
C     ******************************************************************
      PARAMETER (NL=6,NTE=43,NBE=0,NE=43,NSP=700)
      REAL*4 U(NE),X(NE),Y(NE),S(NE),GN(NE),H(NE,NE),P(NE),Q(NE),R(NE)
      REAL*4 Z(NE),KX(NSP),KY(NSP),KXP,KYP,NWBP
      INTEGER IP(NSP),BAD(NE)
      PI=.3141593E+01
      OPEN(UNIT=2,FILE='OUTPUT',STATUS='UNKNOWN')
      OPEN(UNIT=4,FILE='ESTIMATE',STATUS='UNKNOWN')
C     ******************************************************************
C     * GENERATE THE GEOMETRY.                                         *
C     ******************************************************************
      DS=0.6
      BAD(1)=15
      BAD(2)=17
      BAD(3)=20
```

```
      CALL GEOMET(NL,DS,NE,NBE,BAD,X,Y)
C     ******************************************************************
C     * INITIALIZE THE TAPER DISTRIBUTION.                             *
C     ******************************************************************
      IFLAG1=1
      SLL=-40.0
      NBAR=10
      IF (IFLAG1 .EQ. 1) THEN
        CALL TAPER(NE,U,X,Y,SLL,NBAR,NL,DS)
        REWIND 4
        WRITE(4,101) (I,U(I),I=1,NE)
      ELSE
      END IF
      REWIND 4
      READ(4,101) (I,U(I),I=1,NE)
C     ******************************************************************
C     * COMPUTE THE GAIN (DIRECTIVITY) OF THE ARRAY.                   *
C     ******************************************************************
C      CALL GAIN(NE,U,X,Y,GDB)
C     ******************************************************************
C     * FIND THE HALF POWER NULL WIDTH BETWEEN PEAKS.                  *
C     ******************************************************************
C      CALL BEAM(NE,U,X,Y,NWBP)
C     ******************************************************************
C     * GENERATE THE K-SPACE SAMPLE POINTS.                            *
C     ******************************************************************
      BBF=1.24
      CALL KSPACE(NMP,NL,DS,SLL,NBAR,BBF,KX,KY)
C     ******************************************************************
C     * COMPUTE NORMALIZED PEAK SIDELOBE POWER IN DB.                  *
C     ******************************************************************
      CALL PEKPOW(NE,U,X,Y,KXP,KYP,PO)
      NP=0
      CALL PEAKSL(NE,U,X,Y,NSP,NMP,KX,KY,IP,NP,PMAX)
      PMAXDB=10.0*ALOG10(PMAX/PO)
C     ******************************************************************
C     * PERFORM CONJUGATE GRADIENT STEPS.                              *
C     ******************************************************************
      NS=150
      NI=3
      CALL CGRAD(NE,NSP,NMP,NS,NI,U,X,Y,S,NP,IP,KX,KY,GN,H,P,Q,R,Z)
      REWIND 4
      WRITE(4,101) (I,U(I),I=1,NE)
C     ******************************************************************
C     * POWER PATTERN GRAPHICS OUTPUT.                                 *
C     ******************************************************************
      REWIND 4
      READ(4,101) (I,U(I),I=1,NE)
C       ****************************************************************
```

```
C         * PRINT THE POWER PATTERN IN CYLINDRICAL COORDINATES.          *
C         ****************************************************************
           CALL POWPAT(NE,U,X,Y)
C         ****************************************************************
C         * PRINT THE POWER PATTERN IN SPHERICAL COORDINATES.            *
C         ****************************************************************
           CALL SPOWPAT(NE,U,X,Y)
C     *******************************************************************
C     * FORMATS.                                                        *
C     *******************************************************************
  100  FORMAT(I5,F15.5,1X,F15.5)
  101  FORMAT(I5,F15.7)
       END
C
       SUBROUTINE GEOMET(NL,S,NE,NBE,BAD,X,Y)
C     *******************************************************************
C     * COMPUTE THE POSITION OF EACH ELEMENT IN A HEXOGONAL ARRAY.      *
C     *******************************************************************
       REAL*4 X(NE),Y(NE)
       INTEGER BAD(NE),IFLAG
       HGT=(SQRT(3.0)/2.0)*S
C     *******************************************************************
C     * CENTER ROW.                                                     *
C     *******************************************************************
       K=0
       L=0
       XMIN=-NL*S
       DO 1 I=0,2*NL
        XX=XMIN+I*S
        IF (XX .GT. 0.001) THEN
          L=L+1
          CALL CHECK(L,NE,NBE,BAD,IFLAG)
          IF (IFLAG .EQ. 0) THEN
            K=K+1
            X(K)=XX
            Y(K)=0.0
          ELSE
          END IF
        ELSE
        END IF
    1  CONTINUE
C     *******************************************************************
C     * TOP ROWS.                                                       *
C     *******************************************************************
       INUM=2*NL+1
       DO 2 J=1,NL
         INUM=INUM-1
         XMIN=-NL*S+J*S/2.0
         DO 3 I=0,INUM-1
```

```
             XX=IMIN+I*S
             IF (XX .GT. 0.001) THEN
               L=L+1
               CALL CHECK(L,NE,NBE,BAD,IFLAG)
               IF (IFLAG .EQ. 0) THEN
                 K=K+1
                 X(K)=XX
                 Y(K)=HGT*J
               ELSE
               END IF
             ELSE
             END IF
    3      CONTINUE
    2    CONTINUE
C   *******************************************************************
C   * BUTTOM ROWS.                                                    *
C   *******************************************************************
         INUM=2*NL+1
         DO 4 J=1,NL
           INUM=INUM-1
           IMIN=-NL*S+J*S/2.0
           DO 5 I=0,INUM-1
             XX=IMIN+I*S
             IF (XX .GT. 0.001) THEN
               L=L+1
               CALL CHECK(L,NE,NBE,BAD,IFLAG)
               IF (IFLAG .EQ. 0) THEN
                 K=K+1
                 X(K)=XX
                 Y(K)=-HGT*J
               ELSE
               END IF
             ELSE
             END IF
    5      CONTINUE
    4    CONTINUE
         RETURN
         END
C
         SUBROUTINE CHECK(L,NE,NBE,BAD,IFLAG)
C   *******************************************************************
C   * CHECK TO SEE IF THAT ELEMENT IS BAD.                            *
C   *******************************************************************
         INTEGER BAD(NE)
         IFLAG=0
         DO 1 I=1,NBE
            IF (L .EQ. BAD(I)) THEN
               IFLAG=1
            ELSE
```

```
           END IF
 1      CONTINUE
        RETURN
        END
C
        SUBROUTINE BEAM(NE,U,X,Y,BWFN)
C     ****************************************************************
C     * COMPUTE THE BEAM WIDTH BETWEEN FIRST NULLS.                  *
C     ****************************************************************
        REAL U(NE),X(NE),Y(NE),KXI,KYI,KXP,KYP
        RAD=.17453293E-01
        TP=.6283185E+01
        TMIN=0.0
        TMAX=12.0
        P=0.0
        NT=140
        DT=(TMAX-TMIN)/NT
C     ****************************************************************
C     * FIND THE MAXIMUM VALUE.                                      *
C     ****************************************************************
        CALL PEKPOW(NE,U,X,Y,KXP,KYP,PEAK)
C     ****************************************************************
C     * RECOMPUTE AND NOW FIND THE POINT WHICH IS AT THE SLL BELOW THE *
C     * PEAK.                                                        *
C     ****************************************************************
        DO 3 I=0,NT
          T=TMIN+I*DT
          TPS=TP*SIN(RAD*T)
          KXI=TPS
          CALL POWI(NE,U,X,Y,KXI,0.0,POW)
          IF (POW/PEAK .GE. 0.5) THEN
            WRITE(2,100) T,10.0*ALOG(POW/PEAK)
            GO TO 99
          ELSE
          END IF
  3     CONTINUE
  100   FORMAT(F10.5,1X,F10.5)
 99     CONTINUE
        BWFN=2.0*T
        RETURN
        END
C
        SUBROUTINE KSPACE(NMP,NL,DS,SLL,NBAR,BBF,KX,KY)
C     ****************************************************************
C     * THIS SUBROUTINE SAMPLES THE KX >0 REGION OF K SPACE.         *
C     ****************************************************************
        REAL*4 KX(*),KY(*),KC,KR,KXX,KYY,KXMIN,KXMAX,KYMIN,KYMAX,KNULL
        RAD=.17453293E-01
        PI=.3141593E+01
```

```
      TP=.6283185E+01
C     *******************************************************************
C     * COMPUTE THE FIRST ZERO OF THE THEORETICAL BAYLISS PATTERN.      *
C     *******************************************************************
      S=ABS(SLL)
      C1=3.038753E-01
      C2=5.042922E-02
      C3=-2.7989E-04
      C4=3.43E-06
      C5=-2.0E-8
      A=C1+S*(C2+S*(C3+S*(C4+S*C5)))
      C1=9.858302E-01
      C2=3.33885E-02
      C3=1.4064E-04
      C4=-1.9E-06
      C5=1.0E-08
      ZETA1=C1+S*(C2+S*(C3+S*(C4+S*C5)))
      U1=(NBAR+0.5)*SQRT(ZETA1*ZETA1/(A*A+NBAR))
C     *******************************************************************
C     * COMPUTE THE K VALUE OF THE FIRST NULL.                          *
C     *******************************************************************
      R=NL*DS
      KNULL=U1/R
C     *******************************************************************
C     * COMPUTE THE MAXIMUM K VALUE ALLOWED FOR THE FIRST NULL.         *
C     *******************************************************************
C      KC=TP*0.45
      KC=BBF*KNULL
C     *******************************************************************
C     * GENERATE THE RECTANGULAR LATTICE OF SAMPLE POINTS.              *
C     *******************************************************************
      KYMIN=-TP
      KYMAX=TP
      KXMIN=0.0
      KXMAX=TP
      NKX=27
      NKY=45
      DKX=(KXMAX-KXMIN)/(NKX-1)
      DKY=(KYMAX-KYMIN)/(NKY-1)
      NMP=0
      DO 1 I=0,NKY
        KYY=KYMIN+I*DKY
        DO 2 J=0,NKX
          KXX=KXMIN+J*DKX
          KR=SQRT(KXX*KXX+KYY*KYY)
          IF ((KR .GT. KC) .AND. (KR .LT. TP)
     &                                  .AND. (KXX .GT. 0.0)) THEN
            NMP=NMP+1
            KX(NMP)=KXX
```

```
            KY(NMP)=KYY
          ELSE
          END IF
    2   CONTINUE
    1  CONTINUE
       RETURN
       END
C
       SUBROUTINE TAPER(NE,U,X,Y,SLL,NBAR,NL,DS)
C     ******************************************************************
C     * INITIALIZE THE AMPLITUDE DISTRIBUTION.                         *
C     * NOTE THAT THE AMPLITUDE=U*U. THE BAYLISS DISTRIBUTION FORMULAS *
C     * ARE TAKEN FROM MODERN ANTENNA DESIGN BY T. MILLIGAN.           *
C     ******************************************************************
       REAL*4 U(NE),X(NE),Y(NE),ROOT(20),ZETA(4),UZER(20),B(20)
       PI=.3141593E+01
C     ******************************************************************
C     * DEFINE THE ROOTS.                                              *
C     ******************************************************************
       ROOT(1)=0.5860670
       ROOT(2)=1.6970509
       ROOT(3)=2.7171939
       ROOT(4)=3.7261370
       ROOT(5)=4.7312271
       ROOT(6)=5.7345205
       ROOT(7)=6.7368281
       ROOT(8)=7.7385356
       ROOT(9)=8.7398505
       ROOT(10)=9.740895
       ROOT(11)=10.7417435
       ROOT(12)=11.7424475
       ROOT(13)=12.7430408
       ROOT(14)=13.7435477
       ROOT(15)=14.7439856
       ROOT(16)=15.7443679
       ROOT(17)=16.7447044
       ROOT(18) 17.7450030
       ROOT(19)=18.7452697
       ROOT(20)=19.7455093
C     ******************************************************************
C     * COMPUTE APPROXIMATE ARRAY RADIUS.                              *
C     ******************************************************************
       RADIUS=DS*NL
C     ******************************************************************
C     * CHOOSE SIDELOBE LEVEL.                                         *
C     ******************************************************************
       CALL COEFF(SLL,A,ZETA)
       CALL NEWZ(A,ZETA,NBAR,ROOT,UZER)
       CALL FBCOF(ROOT,UZER,NBAR,B)
```

```
C     ******************************************************************
C     * COMPUTE AMPLITUDE.                                             *
C     ******************************************************************
      DO 1 K=1,NE
       R=SQRT(X(K)*X(K)+Y(K)*Y(K))
       COSPHI=X(K)/R
       SUM=0.0
       DO 2 M=1,NBAR
         RHO=R/RADIUS
         ARG=PI*ROOT(M)*RHO
         CALL J1(ARG,BESS1)
         SUM=SUM+B(M)*BESS1
2      CONTINUE
       AMP=COSPHI*SUM
       U(K)=AMP
1     CONTINUE
      CALL NORMAL(NE,U)
      DO 3 I=1,NE
      WRITE(2,*) I,X(I),Y(I),R,RADIUS,U(I)
 3    CONTINUE
100   FORMAT(I5,1X,F10.4,1X,F10.4,1X,F10.4)
      RETURN
      END
C
      SUBROUTINE COEFF(SLL,A,ZETA)
      REAL*4 ZETA(4)
      S=ABS(SLL)
      C1=3.038763E-01
      C2=5.042922E-02
      C3=-2.7989E-04
      C4=3.43E-06
      C5=-2.0E-8
      A=C1+S*(C2+S*(C3+S*(C4+S*C5)))
      C1=9.858302E-01
      C2=3.33865E-02
      C3=1.4064E-04
      C4=-1.9E-08
      C5=1.0E-08
      ZETA(1)=C1+S*(C2+S*(C3+S*(C4+S*C5)))
      C1=2.00337487
      C2=1.141548E-02
      C3=4.159E-04
      C4=-3.73E-06
      C5=1.0E-08
      ZETA(2)=C1+S*(C2+S*(C3+S*(C4+S*C5)))
      C1=3.00636321
      C2=6.83394E-03
      C3=2.9281E-04
      C4=-1.61E-06
```

```
      ZETA(3)=C1+S*(C2+S*(C3+S*C4))
      C1=4.00518432
      C2=5.01795E-03
      C3=2.1735E-04
      C4=-8.8E-07
      ZETA(4)=C1+S*(C2+S*(C3+S*C4))
      RETURN
      END
C
      SUBROUTINE NEWZ(A,ZETA,NBAR,ROOT,UZER)
      REAL*4 ZETA(4),ROOT(20),UZER(20)
      DO 1 N=1,4
        ARG=ZETA(N)*ZETA(N)/(A*A+NBAR*NBAR)
        UZER(N)=ROOT(NBAR+1)*SQRT(ARG)
1     CONTINUE
      DO 2 N=5,NBAR-1
        ARG=(A*A+N*N)/(A*A+NBAR*NBAR)
        UZER(N)=ROOT(NBAR+1)*SQRT(ARG)
2     CONTINUE
      RETURN
      END
C
      SUBROUTINE FBCOF(ROOT,UZER,NBAR,B)
      REAL*4 ROOT(20),UZER(20),B(20)
      REAL*8 RAT,UN2,RM2,RN2,PR1,PR2
      PI=3.141593
      DO 1 M=1,NBAR
        RM2=ROOT(M)*ROOT(M)
        PR1=1.0
        DO 2 N=1,NBAR-1
          UN2=UZER(N)*UZER(N)
          RAT=1.0-RM2/UN2
          PR1=PR1*RAT
2       CONTINUE
        PR2=1.0
        DO 3 N=1,NBAR
          IF (N .NE. M) THEN
            RN2=ROOT(N)*ROOT(N)
            RAT=1.0-RM2/RN2
            PR2=PR2*RAT
          ELSE
          END IF
3       CONTINUE
        ARG=PI*ROOT(M)
        CALL J1(ARG,BESS1)
        B(M)=(RM2/BESS1)*(PR1/PR2)
1     CONTINUE
      RETURN
      END
```

```
C
      SUBROUTINE J1(X,BESS1)
C**********************************************************************C
C THIS SUBROUTINE COMPUTES J1(X)                                       C
C**********************************************************************C
      A0=-0.56249985
      A1=0.21093573
      A2=-0.03954289
      A3=0.00443319
      A4=-0.00031761
      A5=0.00001109
      B0=0.79788456
      B1=0.00000156
      B2=0.01659667
      B3=0.00017105
      B4=-0.00249511
      B5=0.00113653
      B6=-0.00020033
      C0=-2.35619449
      C1=0.12499612
      C2=0.00005650
      C3=-0.00637879
      C4=0.00074348
      C5=0.00079824
      C6=-0.00029166
      IF ((X .GE. 0.0) .AND. (X .LE. 3.0)) THEN
        S=(X/3.0)*(X/3.0)
        P1=S*(A0+S*(A1+S*(A2+S*(A3+S*(A4+A5*S)))))
        BESS1=X*(0.5+P1)
      ELSE IF (X .GT. 3.0) THEN
        T=3.0/X
        Q1=B0+T*(B1+T*(B2+T*(B3+T*(B4+T*(B5+B6*T)))))
        V1=X+C0+T*(C1+T*(C2+T*(C3+T*(C4+T*(C5+C6*T)))))
        BESS1=Q1*COS(V1)/SQRT(X)
      ELSE
      END IF
      RETURN
      END
C
      SUBROUTINE CGRAD(NE,NSP,NMP,NS,NI,U,X,Y,S,NP,IP,KX,KY,GN,H,P,Q,R
     &,Z)
C     *****************************************************************
C     * COMPUTE POWER IN A SINGLE DIRECTION.                          *
C     *****************************************************************
      REAL*4 U(NE),X(NE),Y(NE),S(NE),GN(NE),H(NE,NE),P(NE),Q(NE),R(NE)
      REAL*4 Z(NE),KX(NSP),KY(NSP),KXI,KYI,KXP,KYP
      INTEGER IP(NSP)
C     *****************************************************************
C     * INITIALIZE THE NUMBER OF SIDELOBE SAMPLE POINTS.              *
```

```
C     ****************************************************************
      NP=0
C     ****************************************************************
C     * PERFORM NS RE-STARTS.                                        *
C     ****************************************************************
      DO 1 L=1,NS
C         ************************************************************
C         * FIND THE PEAK SIDELOBE POWER POINT AND UPDATE THE NUMBER *
C         * OF MATCH POINTS.                                         *
C         ************************************************************
          CALL PEAKSL(NE,U,X,Y,NSP,NMP,KX,KY,IP,NP,PMAX)
C         ************************************************************
C         * FIND THE POWER IN THE MAIN BEAM.                         *
C         ************************************************************
          CALL PEKPOW(NE,U,X,Y,KXP,KYP,PEAK)
          PO=PEAK
C         ************************************************************
C         * COMPUTE AND STORE AN ARRAY OF THE 1ST DERIVATIVE OF THE  *
C         * POWER IN THE MAIN BEAM DIRECTION WITH RESPECT TO THE MTH *
C         * VARIABLE.                                                *
C         ************************************************************
          DO 2 M=1,NE
            CALL DPOWI(NE,U,X,Y,M,KXP,KYP,DPWI)
            S(M)=DPWI
2         CONTINUE
C         ************************************************************
C         * COMPUTE AND STORE THE AVERAGE SIDELOBE POWER.            *
C         ************************************************************
          PA=0.0
          DO 3 I=1,NP
            CALL POWI(NE,U,X,Y,KX(IP(I)),KY(IP(I)),POWERI)
            PA=PA+POWERI
3         CONTINUE
          PA=PA/NP
          WRITE(*,200) L,10.0*ALOG10(PA/PO),10.0*ALOG10(PMAX/PO)
          WRITE(2,200) L,10.0*ALOG10(PA/PO),10.0*ALOG10(PMAX/PO)
200       FORMAT(1X,I5,1X,'AVE PEAK SLL',1X,F15.6,1X,
     &                               'PEAK SLL',1X,F15.6)
C         ************************************************************
C         * COMPUTE THE NEGATIVE OF THE GRADIENT.                    *
C         ************************************************************
          DO 4 M=1,NE
C             ********************************************************
C             * COMPUTE THE DERIVATIVE OF THE AVERAGE SIDELOBE POWER. *
C             ********************************************************
              DPAM=0.0
              DO 5 I=1,NP
                CALL DPOWI(NE,U,X,Y,M,KX(IP(I)),KY(IP(I)),DPWI)
                DPAM=DPAM+DPWI
```

```
5             CONTINUE
              DPAM=DPAM/NP
C             ***********************************************************
C             * COMPUTE THE NEGATIVE GRADIENT.                          *
C             ***********************************************************
              GN(M)=(PA*S(M)/PO-DPAM)/PO
4         CONTINUE
C         ***************************************************************
C         * COMPUTE THE HESSIAN.                                        *
C         ***************************************************************
          DO 6 M=1,NE
            DO 7 N=M,NE
              CALL DDPOWI(NE,U,X,Y,M,N,KXP,KYP,DDPWI)
              DDPO=DDPWI
              DDPA=0.0
              DO 8 I=1,NP
                CALL DDPOWI(NE,U,X,Y,M,N,KX(IP(I)),KY(IP(I)),DDPWI)
                DDPA=DDPA+DDPWI
8             CONTINUE
              DDPA=DDPA/NP
              H(M,N)=(GN(N)*S(M)+GN(M)*S(N)+DDPA-PA*DDPO/PO)/PO
              H(N,M)=H(M,N)
7           CONTINUE
6         CONTINUE
C         ***************************************************************
C         * START THE CONJUGATE GRADIENT ALGORITHM.                     *
C         ***************************************************************
C         ***************************************************************
C         * INITIALIZE THE RESIDUAL.                                    *
C         ***************************************************************
          DO 12 M=1,NE
            R(M)=GN(M)
            Z(M)=0.0
12        CONTINUE
C         ***************************************************************
C         * INITIALIZE SEARCH VECTOR.                                   *
C         ***************************************************************
          CALL MATVEC(NE,H,R,Q)
          CALL NORM22(NE,Q,QN)
          BEO=1.0/QN
          DO 13 I=1,NE
            P(I)=BEO*Q(I)
13        CONTINUE
C         ***************************************************************
C         * PERFORM CONJUGATE GRADIENT ITERATIONS.                      *
C         ***************************************************************
          DO 14 K=1,NI
C         ***************************************************************
C         * UPDATE AMPLITUDE VECTOR AND RESIDUAL.                       *
```

```
C          *************************************************************
                CALL MATVEC(NE,H,P,Q)
                CALL NORM22(NE,Q,QN)
                AK=1.0/QN
                DO 15 I=1,NE
                  Z(I)=Z(I)+AK*P(I)
                  R(I)=R(I)-AK*Q(I)
   15           CONTINUE
                CALL NORM22(NE,R,RN)
                ERR=SQRT(RN)
                WRITE(*,500) L,K,ERR
                WRITE(2,500) L,K,ERR
500             FORMAT(3X,'RESIDUAL',1X,I3,1X,I3,1X,E15.8)
C          *************************************************************
C          * UPDATE SEARCH VECTOR.                                     *
C          *************************************************************
                CALL MATVEC(NE,H,R,Q)
                CALL NORM22(NE,Q,QN)
                BEK=1.0/QN
                DO 16 I=1,NE
                  P(I)=P(I)+BEK*Q(I)
   16           CONTINUE
14              CONTINUE
C          *************************************************************
C          * UPDATE AMPLITUDE VECTOR.                                  *
C          *************************************************************
                DO 17 I=1,NE
                  U(I)=U(I)+Z(I)
   17           CONTINUE
                CALL NORMAL(NE,U)
C          *************************************************************
C          * WRITE OUT THE NEW ESTIMATE TO A FILE.     *
C          *************************************************************
             REWIND 4
             WRITE(4,101) (I,U(I),I=1,NE)
101          FORMAT(I5,F15.7)
           IF (NP .EQ. NMP) THEN
             GO TO 99
           ELSE
           END IF
1          CONTINUE
99         CONTINUE
           RETURN
           END
C
           SUBROUTINE PEAKSL(NE,U,X,Y,NSP,NMP,KX,KY,IP,NP,PMAX)
C          *****************************************************************
C          * THIS SUBROUTINE FINDS THE PEAK SIDELOBE POWER LEVEL COORDINATE *
C          * POINT (KX(LMAX),KY(LMAX)).  IF THE POINT IS NEW THEN IT IS     *
```

```
C     * ACCUMULATED INTO THE POINTER ARRAY IP. THE NUMBER OF POINTS IN  *
C     * THE INTEGER POINTER ARRAY IP IS NP SUCH THAT IP(I),I=1...NP.     *
C     *******************************************************************
      REAL U(NE),X(NE),Y(NE) KX(NSP),KY(NSP)
      INTEGER IP(NSP)
C         ***************************************************************
C         * COMPUTE THE POWER AT EACH POINT AND FIND THE MAX.           *
C         ***************************************************************
          CALL POWI(NE,U,X,Y,KX(1),KY(1),PWI)
          PMAX=PWI
          LMAX=1
          DO 1 I=1,NMP
            CALL POWI(NE,U,X,Y,KX(I),KY(I),PWI)
            IF (PWI .GT. PMAX) THEN
              LMAX=I
              PMAX=PWI
            ELSE
            END IF
1         CONTINUE
C         ***************************************************************
C         * COMPARE LMAX TO ALL THE PREVIOUS POINTERS IP(I),I=1...NP.   *
C         * IF LMAX MATCHES ONE OF THE PREVIOUS POINTERS THEN THE       *
C         * POINTER ARRAY NEED NOT BE UPDATED (IFLAG=1).  IF LMAX DOES  *
C         * NOT MATCH A PREVIOUS POINTED VALUE (IFLAG=0) THEN IT        *
C         * BECOMES THE NP+1TH ELEMENT OF THE POINTER ARRAY.            *
C         ***************************************************************
          IFLAG=0
          DO 2 I=1,NP
            IF (IP(I) .EQ. LMAX) THEN
              IFLAG=1
            ELSE
            END IF
2         CONTINUE
          IF (IFLAG .EQ. 0) THEN
            NP=NP+1
            IP(NP)=LMAX
          ELSE
          END IF
      RETURN
      END
C
      SUBROUTINE POWI(NE,U,X,Y,KXI,KYI,POWERI)
C     *******************************************************************
C     * COMPUTE POWER IN A SINGLE DIRECTION.                             *
C     *******************************************************************
      REAL U(NE),X(NE),Y(NE),KXI,KYI
      SUM=0.0
      DO 1 K=1,NE
       PSI=KXI*X(K)+KYI*Y(K)
```

```
      SUM=SUM+U(K)*SIN(PSI)
1    CONTINUE
     POWERI=SUM*SUM
     RETURN
     END
C
     SUBROUTINE DPOWI(NE,U,X,Y,M,KXI,KYI,DPWI)
C    ****************************************************************
C    * COMPUTE 1ST DERIVATIVE OF POWER WITH RESPECT TO THE MTH VARIABLE*
C    * IN THE ITH DIRECTION.                                          *
C    ****************************************************************
     REAL U(NE),X(NE),Y(NE),KXI,KYI
C    ****************************************************************
C    * COMPUTE TI AND VI.                                             *
C    ****************************************************************
     SUM=0.0
     DO 1 K=1,NE
      PSI=KXI*X(K)+KYI*Y(K)
      SUM=SUM+U(K)*SIN(PSI)
1    CONTINUE
C    ****************************************************************
C    * COMPUTE 1ST DERIVATIVE OF TI AND VI.                           *
C    ****************************************************************
     PSIM=KXI*X(M)+KYI*Y(M)
     DPWI=2.0*SIN(PSIM)*SUM
     RETURN
     END
C
     SUBROUTINE DDPOWI(NE,U,X,Y,M,N,KXI,KYI,DDPWI)
C    ****************************************************************
C    * COMPUTE 2ND DERI[illegible] OF POWER WITH RESPECT TO THE M AND NTH  *
C    * VARIABLES IN T[illegible] DIRECTION.                                 *
C    ****************[illegible]*****************************************
     REAL*4 U(NE),[illegible](NE),Y(NE),KXI,KYI
C    ****************************************************************
C    * COMPUTE 2ND DERIVATIVE OF TI AND VI WITH RESPECT TO M,N.       *
C    ****************************************************************
     PSIM=KXI*X(M)+KYI*Y(M)
     PSIN=KXI*X(N)+KYI*Y(N)
C    ****************************************************************
C    * COMPUTE 2ND DERIVATIVE OF POWER.                               *
C    ****************************************************************
     DDPWI=2.0*SIN(PSIM)*SIN(PSIN)
     RETURN
     END
C
     SUBROUTINE NORM22(N,A,AN)
C    ****************************************************************
C    * THIS SUBROUTINE COMPUTES THE EUCLIDIAN NORM SQUARED OF A.      *
```

```
C     *****************************************************************
      REAL*4 A(*)
      AN=0.0
      DO 1 I=1,N
       AN=AN+A(I)*A(I)
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE MATVEC(N,H,P,Q)
C     *****************************************************************
C     * THIS SUBROUTINE COMPUTES THE MATRIX PRODUCT Q=HP.              *
C     *****************************************************************
      REAL*4 H(N,N),P(N),Q(N)
      DO 1 I=1,N
       SUM=0.0
       DO 2 J=1,N
        SUM=SUM+H(I,J)*P(J)
    2  CONTINUE
       Q(I)=SUM
    1 CONTINUE
      RETURN
      END
C
      SUBROUTINE NORMAL(N,A)
C     *****************************************************************
C     * THIS SUBROUTINE NORMALIZES A VECTOR OF LENGTH N.               *
C     *****************************************************************
      REAL*4 A(N)
      VMAX=A(1)
      DO 1 I=1,N
        IF (A(1) .GT. VMAX) THEN
          VMAX=A(I)
        ELSE
        END IF
    1 CONTINUE
      DO 2 I=1,N
       A(I)=A(I)/ABS(VMAX)
    2 CONTINUE
      RETURN
      END
C

      SUBROUTINE PEKPOW(NE,U,X,Y,KXP,KYP,PEAK)
C     *****************************************************************
C     * COMPUTE POWER IN A SINGLE DIRECTION GIVEN A THETA AND PHI.     *
C     *****************************************************************
      REAL U(NE),X(NE),Y(NE),KXI,KYI,KXP,KYP
      RAD=.17453293E-01
```

```
      TP=.6283185E+01
      TMIN=0.0
      TMAX=30.0
      PMIN=-10.0
      PMAX=10.0
      NT=65
      NP=45
      DT=(TMAX-TMIN)/NT
      DP=(PMAX-PMIN)/NP
      DO 1 I=0,NT
        T=TMIN+I*DT
        TPS=TP*SIN(RAD*T)
        DO 2 J=0,NP
          P=PMIN+J*DP
          RP=RAD*P
          KXI=TPS*COS(RP)
          KYI=TPS*SIN(RP)
          CALL POWI(NE,U,X,Y,KXI,KYI,POW)
          IF (I .EQ. 1) THEN
             PEAK=POW
             KXP=KXI
             KYP=KYI
             THEPK=T
             PHIPK=P
          ELSE IF (POW .GT. PEAK) THEN
             PEAK=POW
             KXP=KXI
             KYP=KYI
             THEPK=T
             PHIPK=P
          ELSE
          END IF
 2      CONTINUE
  1   CONTINUE
      WRITE(*,*) THEPK,PHIPK,KXP,KYP,PEAK
      WRITE(2,*) THEPK,PHIPK,KXP,KYP,PEAK
      RETURN
      END
C
      SUBROUTINE GAIN(NE,U,X,Y,GDB)
C     *****************************************************************
C     * COMPUTE POWER IN A SINGLE DIRECTION.                          *
C     *****************************************************************
      REAL U(NE),X(NE),Y(NE),KXP,KYP
      RAD=.17453293E-01
      PI=.3141593E+01
      TP=.6283185E+01
C     *****************************************************************
C     * GRAPH INPUTS                                                  *
```

```
C     ******************************************************************
      NP=41
      NR=35
      PMIN=0.0
      PMAX=360.0
      TMIN=0.0
      TMAX=90.0
      DP=(PMAX-PMIN)/(NP-1)
      DR=(TMAX-TMIN)/(NR-1)
C         **********************************************************
C         * COMPUTE PEAK VALUE.                                    *
C         **********************************************************
           CALL PEKPOW(NE,U,X,Y,KXP,KYP,PEAK)
C         **********************************************************
C         * COMPUTE THE SOLID ANGLE.                               *
C         **********************************************************
           SUM=0.0
           DO 1 I=0,NR-2
             T=TMIN+DR/2.0+I*DR
             ST=SIN(RAD*T)
             DO 2 J=0,NP-2
               P=PMIN+DP/2.0+J*DP
               CALL POWER(NE,U,X,Y,T,P,POW)
               SUM=SUM+ST*POW
2            CONTINUE
1          CONTINUE
           SOLID=RAD*RAD*DR*DP*SUM/PEAK
           GDB=10.0*ALOG10(TP/SOLID)
      RETURN
      END
C
      SUBROUTINE POWPAT(NE,U,X,Y)
C     ******************************************************************
C     * COMPUTE POWER IN A SINGLE DIRECTION.                           *
C     ******************************************************************
      REAL U(NE),X(NE),Y(NE),KXP,KYP
      OPEN(UNIT=7,FILE='DATA')
      RAD=.1745329E-01
      PI=.3141593E+01
      TP=.6283185E+01
C     ******************************************************************
C     * GRAPH INPUTS                                                   *
C     ******************************************************************
      NP=41
      NR=25
      NS=0
      PMIN=0.0
      PMAX=360.0
      TMIN=0.0
```

```
      TMAX=90.0
      DP=(PMAX-PMIN)/(NP-1)
      DR=(TMAX-TMIN)/(NR-1)
      AX=150.0
      AY=AX
      AZ=680.0
      P0=65.0
      Q0=60.0
      XA=-2.0
      XB=2.0
      YA=-2.0
      YB=2.0
      ZA=-2.0
      ZB=2.3
      AT=.3
      WRITE(7,*) NP,NR,NS,AX,AY,AZ
      WRITE(7,*) XA,XB,YA,YB,ZA,ZB
      WRITE(7,*) AT,P0,Q0
C     ******************************************************************
C     * PHI VALUES.                                                    *
C     ******************************************************************
      DO 1 I=0,NP-1
       P=PMIN+I*DP
       WRITE(7,*) RAD*P
 1    CONTINUE
C     ******************************************************************
C     * RADIAL (THETA) VALUES.                                         *
C     ******************************************************************
      DO 2 I=0,NR-1
       T=TMIN+I*DR
       WRITE(7,*) RAD*T
 2    CONTINUE
C     ******************************************************************
C     * FUNCTION VALUES.                                               *
C     ******************************************************************
C          ************************************************************
C          * COMPUTE PEAK VALUE.                                      *
C          ************************************************************
           CALL PEKPOW(NE,U,X,Y,KXP,KYP,PEAK)
C          ************************************************************
C          * WRITE OUT THE NORMALIZED POWER.                          *
C          ************************************************************
           DO 5 I=0,NR-1
             T=TMIN+I*DR
             DO 6 J=0,NP-1
               P=PMIN+J*DP
               CALL POWER(NE,U,X,Y,T,P,POW)
               WRITE(7,*) POW/PEAK
 6           CONTINUE
```

```
    5      CONTINUE
        RETURN
        END
C
        SUBROUTINE SPOWPAT(NE,U,X,Y)
C      *******************************************************************
C      * COMPUTE POWER IN A SINGLE DIRECTION.                            *
C      *******************************************************************
        REAL U(NE),X(NE),Y(NE)
        OPEN(UNIT=8,FILE='SDATA')
        RAD=.17453293E-01
        PI=.3141593E+01
        TP=.6283186E+01
C      *******************************************************************
C      * GRAPH INPUTS                                                    *
C      *******************************************************************
        NP=119
        NR=53
        NS=0
        PMIN=0.0
        PMAX=360.0
        TMIN=0.0
        TMAX=90.0
        DP=(PMAX-PMIN)/(NP-1)
        DR=(TMAX-TMIN)/(NR-1)
        AX=600.0
        AY=AX
        AZ=AX
        PO=65.0
        Q0=60.0
        XA=-2.0
        XB=2.0
        YA=-2.0
        YB=2.0
        ZA=-2.0
        ZB=2.3
        AT=.3
        IT=4
        WRITE(8,*) NP,NR,NS,IT,AX,AY,AZ
        WRITE(8,*) XA,XB,YA,YB,ZA,ZB
        WRITE(8,*) AT,PO,QO
C      *******************************************************************
C      * PHI VALUES.                                                     *
C      *******************************************************************
        DO 1 I=0,NP-1
         P=PMIN+I*DP
         WRITE(8,*) RAD*P
    1   CONTINUE
C      *******************************************************************
```

```
C     * RADIAL (THETA) VALUES.                                              *
C     ***********************************************************************
       DO 2 I=0,NR-1
        T=TMIN+I*DR
        WRITE(8,*) RAD*T
  2    CONTINUE
C     ***********************************************************************
C     * FUNCTION VALUES.                                                    *
C     ***********************************************************************
C          **************************************************************
C          * COMPUTE PEAK VALUE.                                          *
C          **************************************************************
           PEAK=-100.0
           DO 3 I=0,NR-1
             T=TMIN+I*DR
             DO 4 J=0,NP-1
               P=PMIN+J*DP
               CALL POWER(NE,U,X,Y,T,P,POW)
               IF (POW .GT. PEAK) THEN
                  PEAK=POW
               ELSE
               END IF
  4          CONTINUE
  3        CONTINUE
C          **************************************************************
C          * WRITE OUT THE NORMALIZED POWER.                              *
C          **************************************************************
           DO 5 I=0,NR-1
             T=TMIN+I*DR
             DO 6 J=0,NP-1
               P=PMIN+J*DP
               CALL POWER(NE,U,X,Y,T,P,POW)
               WRITE(8,*) POW/PEAK
  6          CONTINUE
  5        CONTINUE
       RETURN
       END
C
       SUBROUTINE POWER(NE,U,X,Y,T,P,POW)
C     ***********************************************************************
C     * COMPUTE POWER IN A SINGLE DIRECTION GIVEN A THETA AND PHI.          *
C     ***********************************************************************
       REAL U(NE),X(NE),Y(NE),KXI,KYI
       RAD=.17453293E-01
       TP=.6283185E+01
       TPS=TP*SIN(RAD*T)
       RP=RAD*P
       KXI=TPS*COS(RP)
       KYI=TPS*SIN(RP)
```

```
      SUM=0.0
      DO 1 K=1,NE
       PSI=KXI*X(K)+KYI*Y(K)
       SUM=SUM+U(K)*SIN(PSI)
1     CONTINUE
      POW=SUM*SUM
      RETURN
      END
```

END

FILMED

DATE: 10-91

DTIC